UDC 531.383
DOI: http://dx.doi.org/10.20535/0203-377130201569529

V. Fedorov[1], *Associate Profesor*, **B. Ivanov**[2], *bachelor*, **A. Olefir**[3], *bachelor*

METHOD OF DETERMINING THE GEOGRAPHIC MERIDIAN PLANE BY GROUND-BASED PENDULUM GYROCOMPASS IN EXPONENTIAL ACCELERATION MODE OF ITS ROTOR

Ua Запропоновано метод визначення положення географічного меридіана триступеневим маятниковим гірокомпасом в процесі експоненційного розгону його ротора.Отримані алгоритми ідентифікації початкового положення вісі ротора, в основу яких покладені як властивості рішень диференційних рівнянь руху гірокомпаса, так і саме рівняння. Розглянуті результати моделювання відповідних алгоритмів обробки інформації. Показані переваги запропонованої методики у порівнянні з традиційними.

Ru Предложен метод определения положения географического меридиана трехстепенным маятниковым гирокомпасом во время процесса экспоненциального разгона его ротора.Получены алгоритмы идентификации начального положения оси ротора, в основу которых положены как свойства решений дифференциального уравнения движения гирокомпаса, так и само уравнение. Рассмотрены результаты моделирования соответствующих алгоритмов обработки информации. Указаны преимуществапредложенной методики по сравнению с традиционными.

Introduction

In carrying out geodetic, topographic, surveying work is an urgent issue of quick and accurate determining of the geographic meridian plane. Having used to solve the problem ground-based pendulum gyrocompasses (GPGC) are composed of high-precision digital sensor of sensitive element (SE) azimuth position, and as a method of determining the plane of the meridian it was used the method of identifying the initial position of the main axis of the device [1] on the results of the analysis of the sensible element motion in the case of rotation of the rotor with a rated angular velocity. With this point of view, the rotor acceleration mode required for bringing the device into operation, is a preparatory, ballast condition, which lengthens the time of measurement significantly. Furthermore, in the case of action of constant uncontrolled moment around a vertical axis of SE, analysis of an azimuth motion by a constant value of rotor rota-

[1] *National Technical University of Ukraine "Kyiv Polytechnic Institute", department of theoretical mechanics*
[2] *National Technical University of Ukraine "Kyiv Polytechnic Institute", Faculty of Heat Power Energy*
[3] *National Technical University of Ukraine "Kyiv Polytechnic Institute", Faculty of Heat Power Energy*

tion speed allows to determine a position of so-called "apparent meridian". This position is displaced from the veritable meridian by an amount proportional to the vertical uncontrolled moment.

In [2, 3] a method of determining the meridian in a condition, when the rotor in a pulse way (for example, using the pyrocatridge) is accelerated to the nominal angular velocity, and then being left to its own, naturally stops is considered. In [4] as a "regular", during which the SE is in a motion in azimuth, the mode of linear increasing of rotor rotation speed is considered. Analysis of the SE motion, which occurs by changing the rotor rotation speed, allows to determine the position of the meridian plane without an error caused by the presence of vertical uncontrolled moment. This is explained by the fact that the "apparent Meridian" changes its position at the time of measurement on the well-known law, determines the change of rotor rotation speed.

Problem formulation

This article discusses the method of determination of the meridian by GPGC, rotor speed of which increases exponentially during measurement. This law mirrors the most accurately the process of natural acceleration of the rotor.

Main body

The system of equations of GPGC's motion when it changes its rotor rotation speed is given by:

$$\begin{gathered} H\dot{\alpha} + HU_{\text{г}}\beta + mgl\beta + HU_{\text{в}} = 0 \\ H\dot{\beta} + \dot{H}\beta - HU_{\text{г}}\alpha = M_0, \end{gathered} \tag{1}$$

where M_0 − permanent uncontrolled moment around the vertical axis of SE,
$H = I\omega$ − the kinetic moment of the gyroscope,
I − axial moment of inertia of the device, ω −the angular velocity of its rotation,
mgl − penduluming of the device;
$U_{\text{г}}$ and $U_{\text{в}}$— horizontal and vertical components of the Earth's angular velocity, respectively;
α and β − rotation angles of SEGPGC in azimuth and vertical plane.

From the system of equations (1) it is easy to obtain the equation of SE motion at azimuthal coordinate. Given $mgl \gg HU_{\text{г}}$ it has the form:

$$H\ddot{\alpha} + 2\dot{H}\dot{\alpha} + mglU_{\text{г}}\alpha = -2\dot{H}U_{\text{в}} - M_0\frac{mgl}{H}. \tag{2}$$

Let the change of kinetic moment occurs according to the law:

$$H = H_m - (H_m - H_0)e^{-\lambda t},$$

where H_0 and H_m - the kinetic moment at the beginning and at the end of acceleration;

λ - attenuation rate of the exponential function.

Let us introduce the independent variable

$$z = (1 - H_0 H_m^{-1}) e^{-\lambda t} (z(0) = z_0 = 1 - H_0 H_m^{-1}).$$

Mark the derivatives of the unknown function on a new variable as α' and α'', write down the equation (2) as follows:

$$z^2(z-1)\alpha'' + z(3z-1)\alpha' - U_\text{г} mgl\lambda^{-2} H_m^{-1}\alpha = \\ = 2U_\text{в}\lambda^{-1} z + M_0 mgl\lambda^{-2} H_m^{-2}(1-z)^{-1}. \quad (3)$$

Supposing [5]:

$$\alpha = z^{i\sqrt{p}} U(z), \quad (4)$$

where $p = U_\text{г} mgl\lambda^{-2} H_m^{-1}$; i - the imaginary unit, write down the equation (3) in the form of:

$$z(z-1)U'' + \left[\left(2i\sqrt{p}+3\right)z - \left(2i\sqrt{p}+1\right)\right]U' + \left(2i\sqrt{p} - p\right)U = \\ = 2U_\text{в}\lambda^{-1} z^{-i\sqrt{p}} + mgl\lambda^{-2} H_m^{-2} M_0 z^{-i\sqrt{p}-1}(1-z)^{-1}. \quad (5)$$

The homogeneous equation corresponding to (5) is an equation of hyper geometric type (Gauss equation) and it has a solution [5]:

$$U(z) = C_1 U_1(z) + C_2 U_2(z), \quad (6)$$

where C_1, C_2 - arbitrary constants,
$U_1(z) = F(a,b,c,z)$ and $U_2(z) = F(a-c+1, b-c+1, 2-c, z)$ – hyper geometric functions with parameters:

$$a = i\sqrt{p};\quad b = i\sqrt{p} + 2;\quad c = 2i\sqrt{p+1}.$$

The particular solution found using the method of variation of arbitrary constants, can be written as:

$$U_p(z) = 2U_\text{в}\lambda^{-1}[U_1(z)Q_2(z) - U_2(z)Q_1(z)] + \\ + mgl\lambda^{-2} H_m^{-2} M_0 [U_1(z)R_2(z) - U_2(z)R_1(z)], \quad (7)$$

where

$$Q_{1,2}(z) = \int \frac{U_{1,2}(z)dz}{z^{1+\alpha}(1-z)\Delta(z)};\quad R_{1,2}(z) = \int \frac{U_{1,2}(z)dz}{z^b(1-z)^2\Delta(z)} \\ \Delta(z) = (1-c)z^{-1}(1-z)^{c-a-b-1}. \quad (8)$$

The general solution of equation (5) consists of the sum of the solution of the homogeneous equation, represented in the form (6), and a particular solution (7). The integration constants C_1 and C_2 are defined by the initial conditions:

$$z = z_0;\ \ U(z_0) = U_0;\ \ U'(z_0) = U_0'\,. \qquad (9)$$

After simple transformations, the solution of equation (5) can be written as

$$\begin{aligned} U(z) = {} & U_0\Delta^{-1}(z_0)\left[U_2{}'(z_0)U_\Gamma(z) - U_1{}'(z_0)U_B(z)\right] - \\ & - U_0{}'\Delta^{-1}(z_0)\left[U_2(z_0)U_\Gamma(z) - U_1(z_0)U_B(z)\right] + \\ & + 2U_B\lambda^{-1}\left[Q_2(z_0,z)U_1(z) - Q_1(z_0,z)U_2(z)\right] + \\ & + mgl\lambda^{-2}H_m^{-2}M_0\left[R_2(z_0,z)U_1(z) - R_1(z_0,z)U_2(z)\right] \end{aligned}$$

where $Q_{1,2}(z_0,z)$, $R_{1,2}(z_0,z)$ –integrals of the form (8) with the limits of integration $z_0$and z.

It is possible to identify from the expression (4) an analytical link between (9) and the initial conditions $\alpha(0) = \alpha_0$ and $\beta(0) = \beta_0$ of the system (1)

$$U_0 = \alpha_0 z_0^{-\alpha},\ \ U_0' = \left[\frac{mgl\beta_0}{H_m\lambda(1 - z_0)} + \frac{U_B}{\lambda} - \alpha\alpha_0\right] z_0^{-(1+\alpha)},$$

Let us write the sought solution of equation (3) of SE motion on the azimuthal coordinate in the form:

$$\begin{aligned} \alpha(z) = {} & \alpha_0 z^\alpha z_0^{-\alpha}\Delta^{-1}(z_0)\left[U_2{}'(z_0)U_1(z) - U_1{}'(z_0)U_2(z)\right] + \\ & + 2U_B\lambda^{-1}z^\alpha\left[U_1(z)Q_2(z_0,z) - U_2(z)Q_1(z_0,z)\right] - \\ & - \left(\frac{z}{z_0}\right)^\alpha \frac{1}{z_0\Delta(z_0)}\left[\frac{mgl\beta_0}{H_m\lambda(1 - z_0)} + \frac{U_B}{\lambda} - \alpha\alpha_0\right]\times \\ & \times\left[U_2(z_0)U_1(z) - U_1(z_0)U_2(z)\right] + \\ & + \frac{mglz^\alpha}{\lambda^2 H_m^2}M_0\left[U_1(z)R_2(z_0,z) - U_2(z)R_1(z_0,z)\right]. \end{aligned}$$

Given the fact that the digital sensor of azimuthal position measured quantity is the angle $(\alpha - \alpha_0)$, as well as the introduction the following designations for the known functions of the parameter z.

$$\begin{aligned} f_1(z) = {} & \left(\frac{z}{z_0}\right)^\alpha \frac{1}{\Delta(z_0)}\left\{\left[U_2{}'(z_0)U_1(z) - U_1{}'(z_0)U_2(z)\right] + \right. \\ & \left. + \frac{\alpha}{z_0}\left[U_2(z_0)U_1(z) - U_1(z_0)U_2(z)\right]\right\} - 1; \end{aligned}$$

$$f_4(z)=(\alpha-\alpha_0)-\frac{z^{\alpha}U_B}{\lambda}\Big\{2\big[U_1(z)Q_2(z_0,z)-U_2(z)Q_1(z_0,z)\big]-$$

$$-\frac{1}{z_0^{1-\alpha}\Delta(z_0)}\big[U_2(z_0)U_1(z)-U_1(z_0)U_2(z)\big]\Big\};$$

$$f_2(z)=-\left(\frac{z}{z_0}\right)^a\frac{1}{z_0\Delta(z_0)}\frac{mgl}{H_m\lambda(1-z_0)}[U_2(z_0)U_1(z)-U_1(z_0)U_2(z)];$$

$$f_3(z)=-\frac{mglz^a}{\lambda^2H_m^2}[U_1(z)R_2(z_0,z)-U_2(z)R_1(z_0,z)].$$

Can be written as: $f_4=\alpha_0f_1+\beta_0f_2+M_0f_3$.

Making changes at discrete moments of time, it is possible, using during the processing of information, such as the method of least squares in its analytic form, to find the best estimates $\widehat{\alpha_{0,}}\,\widehat{\beta_{0,}}\,\widehat{M}$ unknown.

Evaluation $\widehat{\alpha_0}$ provides unambiguous information about the location of the geographic meridian plane. If the analytical solution equation of motion cannot be represented by a linear combination of the unknowns, to find them you can use an algorithm based on the properties of the actual differential equations of gyrocompass's motion [6]. The scheme for obtaining solutions in this case is shown (Fig. 1).

According to (Fig. 1), resulting from the gyrocompass array of information about the SE's azimuthal motion in the rotor acceleration mode is compared with the result of the integration of the differential equations of motion of the device, produced for arbitrary but permissible in terms of the operation of the unit, the initial conditions and perturbations. Then one computes the sum of squared differences of readout a real device $(\alpha-\alpha_0)_n$ and its computer mode $(\alpha-\alpha_0)_m$ (so-called the sum of squares "misalignment"), reduced to one dimension, and exercise machine search for the minimum of this sum. tion $\widehat{\alpha_0}$, corresponding to a minimum sum of squares "misalignment" and it is the "best" estimation of the initial position of the gyrocompass's axis.

With computer simulation of algorithm, block "GYROCOMPASS" (Fig. 1) has been replaced by a block of integration of the differential equation (2), to output of which random noise of a predetermined intensity σ was added.

Computer simulation of the proposed algorithm of the meridian's definition confirmed its efficiency. In Fig. 2 the dependence of error of estimating of the initial position of axis of the gyrocompass's rotor from the time of collection of information for different values of the intensity of the "noise" measurement is shown.

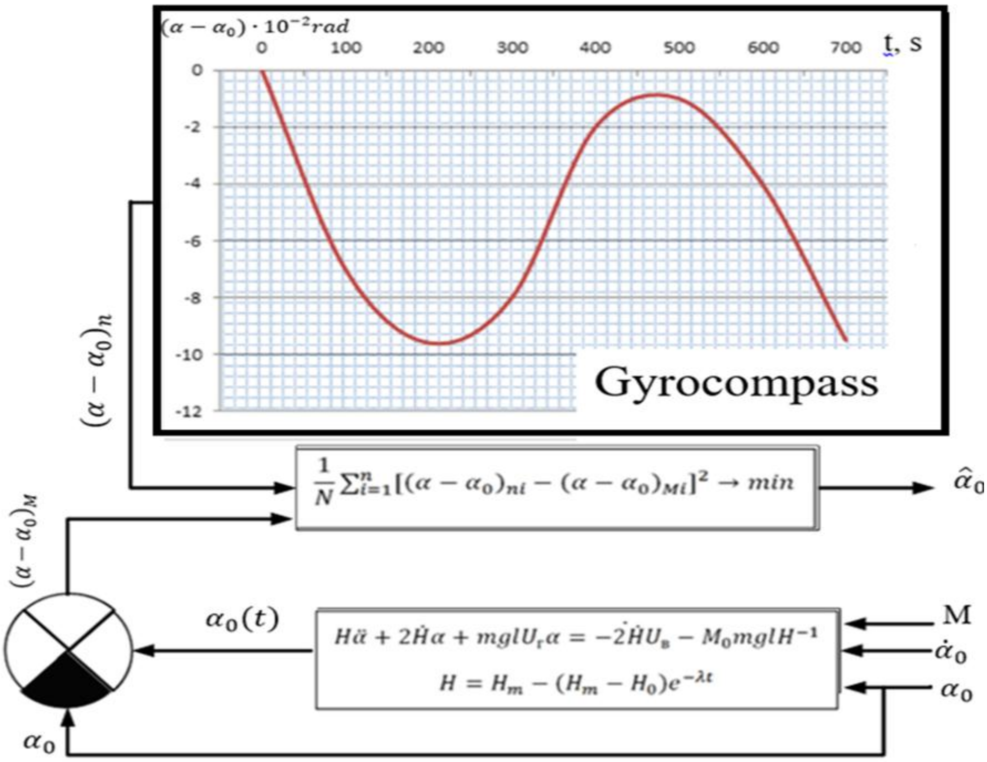

Fig. 1. The scheme of calculating the "best" estimates $\widehat{\alpha_0}$ initial position of gyro rotor axis for exponential acceleration

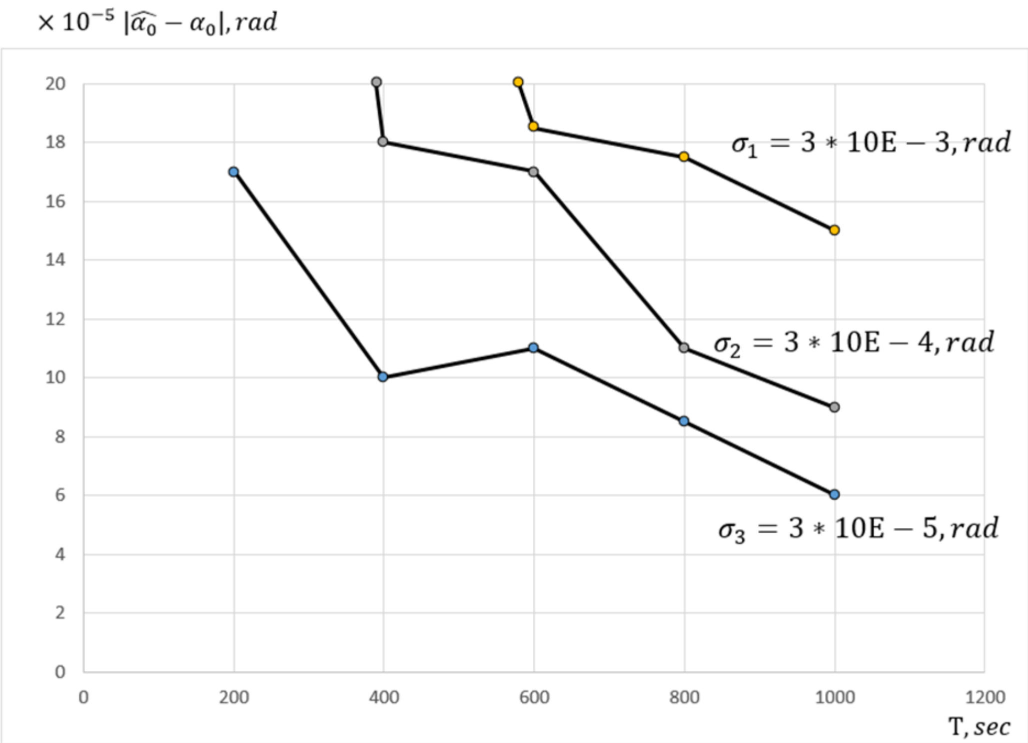

Fig. 2. The dependence of error of estimating of the initial position of axis of the gyrocompass's rotor from the observation time

Conclusions

The article proposes a method of determining the position of geographic meridian by threefold pendulum gyrocompass, based on the analysis of sensitive element azimuthal motion in exponential acceleration mode of its rotor. The scheme identifies the "best" estimation of the initial position of axis of the gyro-compass's rotor, which is based on the differential equations properties of the device's motion.

The proposed method can significantly reduce the time of determination of the meridian, as the set of information is carried out at the stage of the rotor acceleration.

In addition, the results of measurements do not adversely affect the availability of a constant vertical uncontrolled moment, because error caused by this moment is variable over time in and varies according to the known law.

Further investigations that improve the changes of this method are possible in these directions:

1) Studying the sensitivity of the algorithm to the error input a priori data.
2) Compensation of other (except permanent) types of uncontrolled moments that arise during use of the device.

References

1. *Ройтенберг Я. П.* О некоторых косвенных методах получения информации о положении управляемой системы в фазовом пространстве/ Я. П. Ройтенберг// ПММ, 1961. т. ХХУ, вып. 3, с.440-444.
2. *Федоров В. Н.* Алгоритм определения положения географического меридиана трехстепенным маятниковым гирокомпасом во время экспоненциального выбега его ротора/ В. Н. Федоров, Н. И. Штефан// Сборник докладов Х-й Международной научно-технической конференции «Гиротехнологии, навигация, управление движением и конструирование ракетно-космической техники» – Киев, 16–17 апреля 2015. – с.675-680.
3. Патент Украины UA №101962 на полезную модель. Способ определения направления географического меридиана маятниковым гирокомпасом/Федоров В. Н., Штефан Н. И. Заявлено 06.03.2015. Опубликовано 12.10.2015. Бюллетень №19 – 5 с.
4. *Федоров В. Н.* Об определении положения географического меридиана трехстепенным маятниковым гирокомпасом во время разгона его ротора /В. Н. Федоров//Механіка гіроскопічних систем. –2014. –№27. – с. 30-36.
5. *Камке Э.* Справочник по обыкновенным дифференциальным уравнениям/ Э. Камке – М.: Наука, 1971. – 576 с.