UDC 539.3

DOI: https://doi.org/10.20535/0203-3771502025347455

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CALCULATION OF THE CRITICAL MOMENT OF THE WING CONSOLE STABILITY LOSS UNDER AERODYNAMIC LOAD

Ua У зв'язку із тим, що розрахунок аеродинамічних характеристик літака є дуже важливою частиною проєктування літака, виникає необхідність розвивати чисельні та аналітичні методи дослідження коливань та стійкості елементів літака, у тому числі і крила. У роботі розглянуто аналітичний розв'язок задачі стійкості крила літака під дією аеродинамічного навантаження. Отримано значення критичного моменту у разі якого крило втрачає стійкість. Розрахунки були проведені для характеристик такого типового матеріалу для авіабудування як алюміній.

En Due to the fact that the calculation of the aerodynamic characteristics of an aircraft is a very important part of aircraft design, there is a need to develop numerical and analytical methods for studying vibrations and stability of aircraft elements, including the wing. The paper considers an analytical solution to the problem of stability of an aircraft wing under the action of aerodynamic loading. The value of the critical moment at which the wing loses stability is obtained. The calculations were carried out for the characteristics of such a typical material for aircraft construction as aluminum.

Introduction

For reliable and long-term use of the aircraft, it is necessary to calculate the structural elements of the aircraft as accurately as possible, especially under the influence of aerodynamic forces. In this matter, the use of analytical calculation methods is of great importance [1 – 3, 5 – 7]. Currently, the issue of designing an aircraft structural element optimized for the perception of aerodynamic loads, primarily the wing, occupies an important place.

Despite the large number of works on the calculation of aircraft structural elements, the stability of the wing console under the action of aerodynamic loading is not sufficiently presented. It should be emphasized that when designing a wing, it is necessary to prevent destruction or irreversible change in the shape of aircraft elements, the appearance of unacceptably large vibrations and the development of oscillatory instability of the structure, as well as to limit the movement of structural elements [8 – 10].

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During steady flight, an aircraft is subjected to mass (conservative) and aerodynamic (non-conservative) forces. Mass forces include the weight of the aircraft itself, and aerodynamic forces include lift, engine thrust, and aircraft drag [9 – 11].

Nonconservative forces are those whose work on a closed loop is not zero, depends on the point of application of the forces and their trajectory, i.e. the law of conservation of mechanical energy is not fulfilled. This complicates the task of assessing the behavior of the wing under different types of loading. Let us consider the effect of engine thrust on the wing console of an aircraft.

Statement of the problem

To simplify the calculations, we will assume that the wing is a continuous isotropic beam of rectangular cross-section, which is loaded at the end by a concentrated moment L, and the engine itself is integrated into the wing, in order to ignore the torsion from the eccentricity between the center of stiffness of the wing and the direction of the thrust force. When the wing twists, the load also changes direction because the engine moves with the wing.

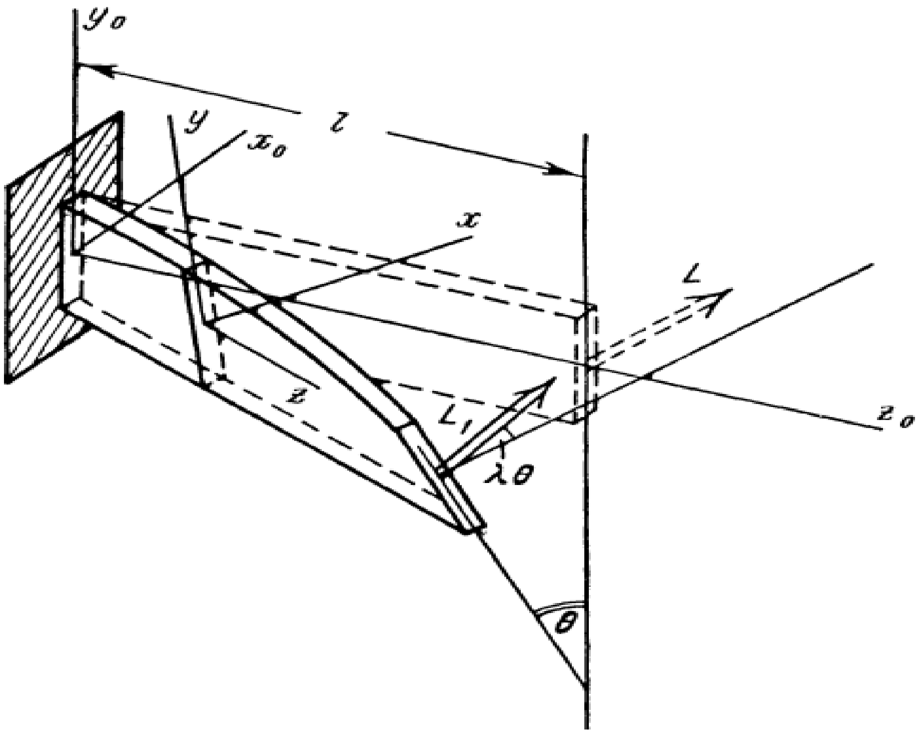

Fig. 1. Scheme of wing deformation under the action of aerodynamic load

Let us introduce a fixed coordinate system x_0 y_0 z_0 . and also a moving system that rotates together with the cross-section of the strip *x y z* (Fig. 1)

Let L be the moment vector before deformation, and L_1 – after deformation. We assume that the vector L_1 is in the plane of the cross-section but with the vector L it forms an angle λ_θ, proportional to the angle of rotation θ of the end section around Oz_0.

Then, projecting the vector L_1 onto the fixed coordinate axes, we obtain with accuracy up to first-order quantities:

$$L_{xo} = L, \quad L_{yo} = L\lambda\theta(l)\ , \quad L_{zo} = -L\frac{du(l)}{dz}\ .$$

For a moving coordinate system, we're find:

$$L_z = L,\ L_y = -L\theta(z) + L\lambda\theta(l),\ L_z = L\frac{du(z)}{dz} - L\frac{du(l)}{dz}.$$

Solution of the problem

Considering the equilibrium position of the wing adjacent to the unperturbed one, we obtain the system of equations

$$\begin{cases} EJ_y\dfrac{d^2u}{dz^2} = -L\theta + L\theta\lambda(l) \\ GJ_d\dfrac{d\theta}{dz} = L\dfrac{du}{dz} - L\dfrac{du(l)}{dz}. \end{cases}$$

This system is equivalent to the equation

$$\frac{d^2\theta}{dz^2} + k^2\theta = k^2\lambda\theta(l),$$

where $k^2 = \dfrac{L^2}{EJ_yGJ_d}$.

The integral of this equation has the form

$$\theta = C_1\sin kz + C_2\cos kz + \theta\lambda(l).$$

Subjecting the integral to the boundary condition of our cantilever fixation, namely:

$$\theta(0) = \frac{d\theta(l)}{dz} = 0.$$

We obtain the characteristic equation:

$$\cos kl = -\frac{\lambda}{1-\lambda}.$$

The equality has real roots as long as $\lambda \le 0,5$. Under this condition, the loss of wing stability occurs by the type of branching of equilibrium forms.

At $\lambda > 0,5$, for any values of the moment L, there are no forms of equilibrium adjacent to the unperturbed one. In this case, the loss of stability can occur only by the type of oscillatory instability.

Let us consider the case when $\lambda = 1$.

The equation of small oscillations has the form [3]:

$$\begin{cases} EJ_y \dfrac{d^4u}{dz^4} + L\dfrac{d^2\theta}{dz^2} + m\dfrac{d^2u}{dt^2} = 0 \\ -GJ_d \dfrac{d^2\theta}{dz^2} + L\dfrac{d^2u}{dz^2} - mr^2\dfrac{d^2\theta}{dt^2} = 0. \end{cases}$$

And the boundary conditions are:

$$\begin{cases} u(0) = \dfrac{du(0)}{dz} = \theta(0) = 0 \\ \dfrac{d^2u(l)}{dz^2} = \dfrac{d^3u(l)}{dz^3} = \dfrac{d\theta(l)}{dz} = 0. \end{cases}$$

We are looking for a solution in the form [3, 4]

$$u(z,t) = U(z)e^{i\Omega t}$$

$$\theta(z,t) = \Theta(z)e^{i\Omega t}$$

$$U(\zeta) = C_1\gamma_1 \mathrm{ch}\mu_1\zeta + C_2\gamma_1 \mathrm{sh}\mu_1\zeta + C_3\gamma_2 \cos\mu_2\zeta + {} \\ + C_4\gamma_2 \sin\mu_2\zeta + C_5\gamma_3 \cos\mu_3\zeta + C_6\gamma_3 \sin\mu_3\zeta,$$

$$\Theta(\zeta) = D_1 \mathrm{ch}\mu_1\zeta + D_2 \mathrm{sh}\mu_1\zeta + D_3 \cos\mu_2\zeta + {} \\ + D_4 \sin\mu_3\zeta + D_5 \cos\mu_3\zeta + D_6 \sin\mu_3\zeta.$$

where

$$\left.\begin{aligned} &\mu_{1,2} = \sqrt{\pm\frac{1}{2} + \sqrt{\frac{1}{4} + \omega^2}},\mu_3 = ns, \\ &\gamma_1 = -\beta\pi l\sqrt{\frac{GJ_d}{EJ_y}}\frac{\mu_1^2}{\mu_1^2 - n^2} = \frac{l}{\beta\pi}\sqrt{\frac{GJ_d}{EJ_y}}, \\ &\gamma_2 = \beta\pi l\sqrt{\frac{GJ_d}{EJ_y}}\frac{\mu_2^2}{\mu_2^4 - n^2} = \frac{l}{\beta\pi}\sqrt{\frac{GJ_d}{\pi}} = \gamma_1, \\ &\gamma_3 = \beta\pi l\sqrt{\frac{GJ_d}{EJ_y}}\frac{\mu_3^2}{\mu_3^4 - n^2} = -\beta\pi l\sqrt{\frac{GJ_d}{EJ_y}}\,s^2, \\ &\omega^2 = \frac{m}{EJ_y}\left(\frac{l}{\beta\pi}\right)^4\Omega^2, n = \frac{m}{EJ_y}l^4\Omega^2, s = \sqrt{\frac{EJ_y n^2}{GJ_d l^2}}, \beta = \frac{Ml}{\pi\sqrt{EJ_y GJ_d}}. \end{aligned}\right\}$$

Having satisfied the boundary conditions, we obtain the following characteristic equation

$$\Delta\left(\omega^2,\beta\right)=\gamma_1^2\gamma_3\left(\gamma_1-\gamma_3\right)\mu_1\left(\mu_1^2+\mu_2^2\right)\left(\mu_3\sin\mu_3+\mu_1\,\mathrm{sh}\,\mu_1\right)\mathrm{sh}\,\mu_1\cos\mu_2-$$
$$-\gamma_1^2\left(\mu_2\sin\mu_2+\mu_1\,\mathrm{sh}\,\mu_1\right)\times\cos\mu_3\left[\mu_2\left(\gamma_1-\gamma_3\right)\left(\gamma_3\mu_3^2+\gamma_1\mu_1^2\right)\mathrm{sh}\mu_1\right]+$$
$$+\gamma_1^2\gamma_3\left(\gamma_1-\gamma_3\right)\mu_2\left(\mu_1^2+\mu_2^2\right)\left(\mu_3\sin\mu_3-\mu_2\sin\mu_2\right)\sin\mu_2\times$$
$$\times\,\mathrm{ch}\,\mu_1-\left[\gamma_1\mu_2^2\left(\gamma_1-\gamma_3\right)\cos\mu_2+\gamma_1\mu_2^2\left(\gamma_1-\gamma_3\right)\mathrm{ch}\,\mu_1\right]\times$$
$$\times\left[\gamma_1\left(\gamma_1\mu_1^2-\gamma_3\mu_3^2\right)\cos\mu_2\cos\mu_3+\gamma_1\left(\gamma_3\mu_3^2+\gamma_1\mu_1^2\right)\mathrm{ch}\,\mu_1\cos\mu_3-\right.$$
$$\left.-\gamma_1\gamma_3\left(\mu_1^2+\mu_2^2\right)\mathrm{ch}\,\mu_1\cos\mu_3\right]=0\,.$$

For a sufficiently narrow and long beam, which is what a wing is, we can assume that

$$\delta/h\le 1/10, h/l\le 1/20,$$

then

$$s=\sqrt{\frac{EJ_y r^2}{GJ_d l^2}}\sim 10^{-2}\,.$$

What makes it possible $\gamma_3\ll\gamma_1$, and therefore the characteristic equation is significantly simplified.

Calculations were carried out for the characteristics of such a typical material for aircraft construction as aluminum with the characteristics

Elastic modulus $E=7\cdot 10^4\,MPa$,

Poisson's ratio $\mu\;=0{,}34$,

Shear modulus $G=2{,}6\cdot 10^4\,MPa$.

A graph of the dependence between the solutions of the characteristic equation on the parameter β is constructed.

It is clearly seen from the graph that the critical value of the parameter β is approximately equal to 1,43.

And with the definition of β we can calculate the critical value of the moment L which is equal to M*

$$M^*=1{,}43\frac{\pi\sqrt{EJGJ_d}}{l}\,.$$

At this value of the moment M*, the wing will lose stability.

Knowing the actual value of the bending moment created by the engine and the critical value of the moment, which depends on the geometric characteristics of the wing, you can protect it from oscillations and subsequent problems that arise with this.

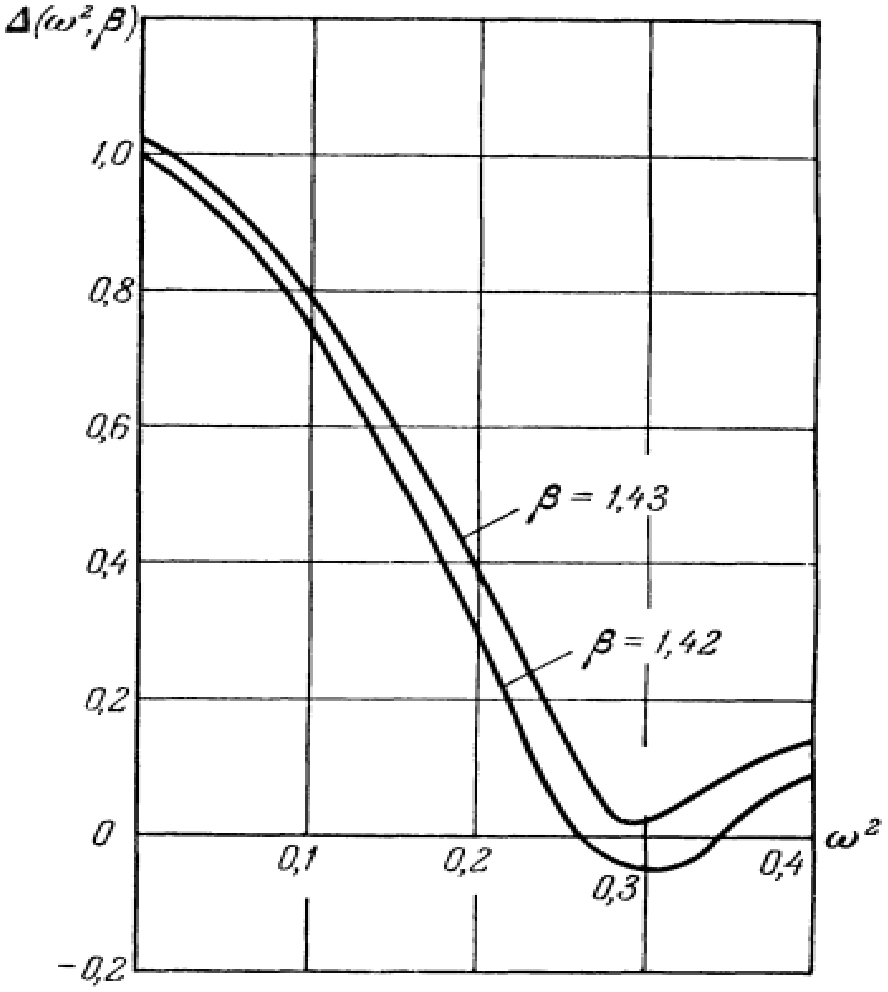

Fig. 2. Graph of the dependence of the solutions of the characteristic equation on the geometric parameters of the wing model

Conclusions

The paper presents an analytical solution to the problem of stability of an aircraft wing under the action of a non-conservative load. The characteristic equation is determined analytically from the differential equations of small oscillations and the critical value of the moment of loss of stability of the aircraft console is calculated, which depends on the geometric and physical and mechanical characteristics of the structure. Calculations were carried out for a wing made of aluminum.

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