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OSCILLATIONS OF A NONLINEAR ASYMMETRIC MECHANICAL SYSTEM IN THE NEAR-RESONANT REGION

Introduction

A considerable number of studies in the field of dynamic of mechanical systems are devoted to the studying of systems' oscillations with symmetric characteristic of the restoring force [1–2].

One of the main problems in the studying of the dynamic behavior of nonlinear systems is the reliable determination of their resonant frequencies. Notions about the resonance phenomena in nonlinear systems are based mainly on mathematical calculations, and declared theoretically resonant frequencies don't always match the computing and natural experiments in practice [1].

Therefore, is need to find more signs of resonance phenomena, which would manifest the same, both in linear and nonlinear systems, for reliable determination of the resonance frequencies of nonlinear systems.

At the same time, many systems have asymmetric vibrational characteristics of the restoring force [3]. Therefore, the development of methods for constructing solutions of nonlinear equations in the resonant and near-resonant fields with and without energy dissipation is an important task of mechanics.

Statement and solution of the problem

Let us consider an oscillating system, which has an asymmetrical respect to the origin of the restoring force characteristic [4].

The equation of oscillations of such a system with one degree of freedom can be written as:

$$\ddot{q} + \omega_0^2\left(1 - \bar{b}q\right)q = \varepsilon P_0 \cos pt, \qquad (1)$$

here q –is the generalized coordinate which is measured from the equilibrium position;

$\omega_0^2 = c/a$– is the squared natural frequency;

$\bar{b}$ – is the coefficient of variation of the restoring force characteristics from the nonlinear dependence;

p –is the frequency of driving force;

$P_0 = P/a$ –is reduced amplitude of the driving force;

a –is the coefficient of inertia with c– stiffness;

ε – is a small parameter; $b = \omega_0^2/\bar{b}$.

We can find only one solution for $p = \omega_0$ as an expansion in degrees of the small parameter $\varepsilon^{(2k=1)^{-1}}$, where $2k$ – is the minimal degree, $k = 1, 2, \ldots, N$, using I. V. Malkin's method [4].

Setting $\varepsilon = 0$, from equation (1) we obtain the equation of the generating system

$$\ddot{q} + \omega_0^2 q - bq^2 = 0 \tag{2}$$

We found the expansion for square of the frequency,using Lyapunov's method [4]:

$$\omega^2 = \omega_0^2 - \frac{5b^2A^2}{\omega_0^2} - \cdots \tag{3}$$

Equation (3) shows that$2k = 2$.

Consequently, the value of the small parameter equal to $\varepsilon^{1/3}$[4];

$$q = \varepsilon^{1/3} q_1 + \varepsilon^{2/3} q_2 + \varepsilon q_3 + \varepsilon^{4/3} q_4 + \cdots \tag{4}$$

It should be noted, that in fact, the nonlinear oscillation systems in the near-resonant field have amplitude which is greater than the resonance [4].

Therefore, it is advisable to explore the near-resonance oscillations of the mechanical system, taking into account the energy dissipation.

Suppose there is a small frequency detuning which is proportional to $\varepsilon^{2/3}$:

$$\alpha = p^2 - \omega_0^2 \tag{5}$$

Imperfect elasticity of the material will considered by the using of Boca-Schlippe-Kolar hypothesis, where $\frac{1}{\omega_0}\frac{\partial}{\partial t}$ – is Boca- Schlippe's operator.

Then loop's characteristic of equation becomes:

$$F(q) = \omega_0^2 q - bq^2 + \beta\omega_0^{-2}|\dot{q}|\dot{q} \tag{6}$$

where

$b = \omega_0^2 \tilde{b}$; $\beta = \omega_0^2 \tilde{\beta}$; $\beta \ll b$;

$$\omega_0^2 \tilde{\beta} \left|\frac{\dot{q}}{\omega_0}\right| \dot{q} = \beta\omega_0^{-2}|\dot{q}|\dot{q} = \tilde{\beta}|\dot{q}|\dot{q} \tag{7}$$

Substituting the values of (5) and (7) in equation (1), we obtain:

$$\ddot{q} + p^2 q - bq^2 + \varepsilon^{1/3}\beta\omega_0^{-2}|\dot{q}|\dot{q} = \varepsilon^{2/3}\alpha q + \varepsilon P_0 \cos pt \tag{8}$$

When $\varepsilon = 0$, equation (8) is transformed into the equation of the generating system (2).

Consequently, the periodic solution can be represented in the form (4). Substituting (4) into (8) and equating coefficients of like powers of ε, we obtain the system of equations:

$$\begin{cases} \ddot{q}_1 + p^2 q_1 = 0; \\ \ddot{q}_2 + p^2 q_2 = b q_1^2; \\ \ddot{q}_3 + p^2 q_3 = 2b q_1 q_2 + \tilde{\beta}|\dot{q}|\dot{q} + \alpha q_1 + P_0 \cos pt. \end{cases} \quad (9)$$

After substitution of the solution of equations (9) in equation (4), we obtain the solution of equation (8):

$$q = A\cos(pt - \varphi) + \frac{b^2 A^2}{2p^2} - \frac{b^2 A^2}{6p^2}\cos 2(pt - \varphi) + \frac{b^2 A^3}{48p^4}\cos 6(pt - \varphi) \quad (10)$$

The main problem for a mechanical system with *N* degrees of freedom, respectively, is arises in determining of the spectrum of natural frequencies and forms of oscillations. The nonlinear vibration systems can be studied after solving this problem and proposed above techniques. To determine the spectrum of natural frequencies and mode shapes of the system with *N* degrees of freedom is proposed to use a method which based on the minimization of the functional type of the Rayleigh by the iterative method of descent [5]. Method of descent is an economical and sustainable in terms of the computational process algorithm and allows us to solve the problem of high dimensionality, using the limited resources of a PC.

Conclusions

We studied the forced oscillations of nonlinear systems with asymmetric characteristic of the restoring force. It is shown that the amplitude of the oscillations at the exact resonance is less than other values of the oscillation amplitude in the near-resonant region. Accounting for energy dissipation in the system was considered on the basis of the hypothesis Boca Schlippe-Kolar. The proposed approach makes it possible to determine the maximum amplitude of the oscillations of a nonlinear system more accurately.

References

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