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DYNAMICS OF TRAJECTORY ROTATION IN CORIOLIS VIBRATORY GYROSCOPES

Introduction

In view of constantly growing market for micromechanical angular rate sensors Coriolis vibratory gyroscopes (CVGs) have received significant amount of attention from the MEMS sensors design specialists due to the promising possibility to fabricate sensitive elements of such gyroscopes in miniature form by using modern microelectronic mass-production technologies. While conventional angular rate measurement is based on detection of the rotation induced oscillations amplitude (secondary amplitude) [1], trajectory analysis approaches are utilised as well [2, 3]. The latter also allows designing rate integrating sensors, which are more suitable for attitude and navigation applications [3, 4]. This paper addresses problems related to modelling angle of the sensitive element motion trajectory rotation due to the presence of the external angular rate.

Problem formulation

It is well known that in general case motion trajectory of the CVG sensitive element is an ellipse. Angle of the trajectory rotation in steady state is proportional to the angular rate. Major goal of this paper is to study dynamics of the sensitive element motion trajectory due to the external angular rate, and to develop its transient process mathematical model. Obtained model can be later used to improve performances of the Coriolis based angular rate sensors.

Sensitive element motion trajectory

One could consider the CVG sensitive element as a two-dimensional pendulum, whose steady state trajectory forms a rotated ellipse, as shown in Fig. 1. In this figure, a and b are the big and small half-axes of the ellipse, θ is the angle of the ellipse rotation relatively to the axes of primary x_1 and secondary x_2 oscillations. It is well-known, that these parameters (namely half-axes and angle of rotation) depend on amplitudes and phases of primary and secondary oscillations, which in turn depend on parameters of the sensitive element design and unknown angular rate. The problem, which is to be addressed in this paper, is to develop and analyse mathematical model of the θ angle transient processes due to the angular rate.

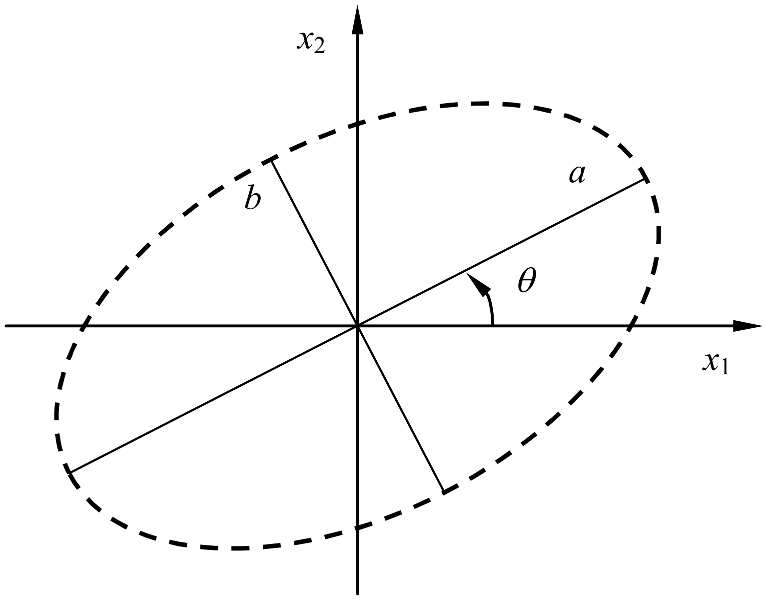

Fig. 1. Sensitive element motion trajectory

In general case, angle of the trajectory rotation is given by the following expression [2]:

$$\theta = \frac{1}{2}\mathrm{tg}^{-1}\left[\frac{2A_1A_2\cos\varphi}{A_1^2 - A_2^2}\right]. \tag{1}$$

Here A_1 and A_2 are the primary and secondary amplitudes of the sensitive element oscillations, φ is the phase shift between the primary and secondary oscillations. In order to calculate these amplitudes as functions of the angular rate, let us analyse dynamics of the CVS sensitive element.

Sensitive element dynamics

In the most generalized form, motion equations of the CVG sensitive element both with translational and rotational motion could be represented in the following form [1]:

$$\begin{cases} \ddot{x}_1 + 2\zeta_1 k_1 \dot{x}_1 + (k_1^2 - d_1\Omega^2)x_1 + g_1\Omega\dot{x}_2 + d_3\dot{\Omega}x_2 = q_1(t), \\ \ddot{x}_2 + 2\zeta_2 k_2 \dot{x}_2 + (k_2^2 - d_2\Omega^2)x_2 - g_2\Omega\dot{x}_1 - \dot{\Omega}x_1 = q_2(t). \end{cases} \tag{2}$$

Here x_1 and x_2 are the generalized coordinates that describe primary (excited) and secondary (sensed) motions of the sensitive element respectively, k_1 and k_2 are the corresponding natural frequencies, ζ_1 and ζ_2 are the dimensionless relative damping coefficients, Ω is the measured angular rate, which is orthogonal to the axes of primary and secondary motions, q_1 and q_2 are the generalized accelerations due to the external forces acting on the sensitive element. The remaining dimensionless coefficients are different for the sensitive elements exploiting either translational or rotational motion. For the translational sensitive element they are $d_1 = d_2 = 1$, $d_3 = m_2/(m_1 + m_2)$, $g_1 = 2m_2/(m_1 + m_2)$, $g_2 = 2$, where were m_1 and m_2 are the masses of the outer frame and the internal massive element.

Steady state solution of the equations (2) in terms of amplitudes and phases of primary and secondary oscillations can be represented as follows:

$$
\begin{gathered}
A_1 = \frac{q_{10}}{k^2\sqrt{(1-\delta\omega^2)^2 + 4\zeta_1^2\delta\omega^2}}, \\
A_2 = \frac{A_1 g_2 \delta\omega}{\sqrt{\delta k^4 + \delta\omega^2 - 2\delta k^2 \delta\omega^2 (1 - 2\zeta_2^2)}}\delta\Omega, \\
\varphi_1 = -\arctan\frac{2\delta\omega\zeta_1}{1-\delta\omega}, \\
\varphi_2 = -\tan^{-1}\frac{\delta k^2 - (1 + 4\zeta_1\zeta_2\delta k + \delta k^2)\delta\omega^2 + \delta\omega^4}{2\delta k\delta\omega(\zeta_2 + \zeta_1\delta k) - 2\delta\omega^3(\zeta_1 + \zeta_2\delta k)}.
\end{gathered} \tag{3}
$$

Here q_{10} is the amplitude of accelerations created by the primary excitation system, k is the primary natural frequency, $\delta\omega = \omega/k$ is the relative excitation frequency, $\delta k = k_2/k_1$ is the ratio of the secondary and primary natural frequencies, $\delta\Omega = \Omega/k$ is the relative angular rate. Angular rate is assumed to be negligible in comparison to the natural frequencies.

One should note that the sensitive element trajectory parameters depend on the phase shift $\varphi = \varphi_2 - \varphi_1$ between primary and secondary phases. Most importantly, based upon (3), phases of do not depend on angular rate. In case of the primary resonance ($\delta\omega = 1$), cosine of this phase shift can be calculated as

$$\cos\varphi = \frac{2\zeta_2\delta k}{\sqrt{\delta k^4 - 2(1-2\zeta_2^2)\delta k^2 + 1}}. \tag{4}$$

Expressions (3) along with the phase shift representations (4) can now be used to analyse parameters of the actual trajectory of the CVG sensitive element. However, dependencies (3) and (4) are the steady state solutions, which do not allow studying transient processes when external angular rate is applied.

Trajectory rotation angle

As has been demonstrated in [5], Laplace transformation of the secondary amplitude with respect to settled primary oscillations is

$$A_2(s) = \frac{q_{10}g_2}{4k_1^2\zeta_1(s + k_2\zeta_2)}\Omega(s). \tag{5}$$

Using expressions (3-5) we can modify expression (1) to the following form

$$\theta(s)=\frac{1}{2}\mathrm{tg}^{-1}\left[\frac{4g_2k(s+k\zeta_2\delta k)\cos\varphi}{[4(s+k\zeta_2\delta k)^2-g_2^2k^2\delta\Omega^2(s)]}\delta\Omega(s)\right]. \tag{6}$$

Apparently expression (6) is non-linear in terms of the input angular rate. However, taking into account that relative angular rate is small ($\delta\Omega << 1$), expression (6) can be linearised with respect to the small $\delta\Omega$ as follows:

$$\theta(s)\approx\frac{g_2k\zeta_2\delta k}{(s+k\zeta_2\delta k)\sqrt{\delta k^4-2(1-2\zeta_2^2)\delta k^2+1}}\delta\Omega(s). \tag{7}$$

Finally, assuming matching natural frequencies of primary and secondary oscillations ($\delta k = 1$), expression (7) can be further simplified to

$$\theta(s)\approx\frac{g_2k}{2(s+k\zeta_2)}\delta\Omega(s). \tag{8}$$

Steady state of the obtained expression (8) is in perfect agreement with the previously published steady state expressions for the motion trajectory angle of rotation [2].

Corresponding to (7) and (8) transfer functions from the relative angular rate to the trajectory rotation angle are as follows:

$$W_\theta^\Omega(s)=\frac{\theta(s)}{\delta\Omega(s)}\approx\frac{g_2k\zeta_2\delta k}{(s+k\zeta_2\delta k)\sqrt{\delta k^4-2(1-2\zeta_2^2)\delta k^2+1}}\approx\frac{g_2k}{2(s+k\zeta_2)}. \tag{9}$$

Transfer functions (9) can now be used to synthesise systems to control sensitive element motion trajectory as well as to implement advanced methods of the angular rate measurements.

Numerical simulations

Numerical simulation of the sensitive element motion trajectory based on the equations (2) is shown in Fig. 2.

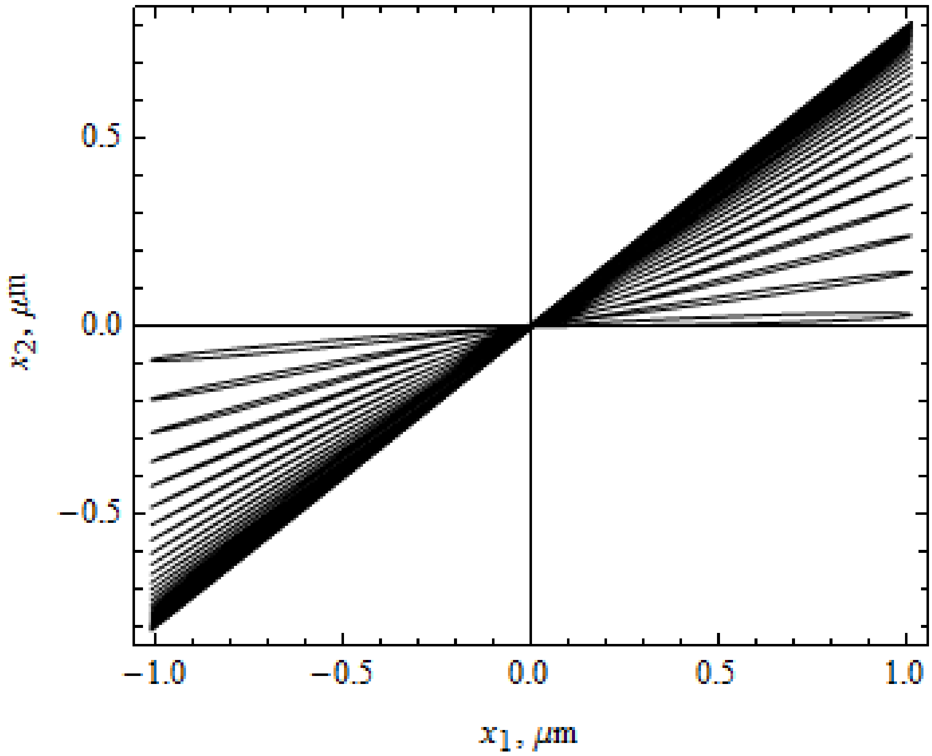

Fig. 2. CVG sensitive element motion trajectory

Primary oscillations are assumed to be already settled and constant angular rate is applied. Corresponding simulations for the angle of trajectory rotation are shown in Fig. 3.

Here dashed line corresponds to the simplified approximation (8). One can see that significant steady state error is present, which reduced usability of the derived simplified model (8).

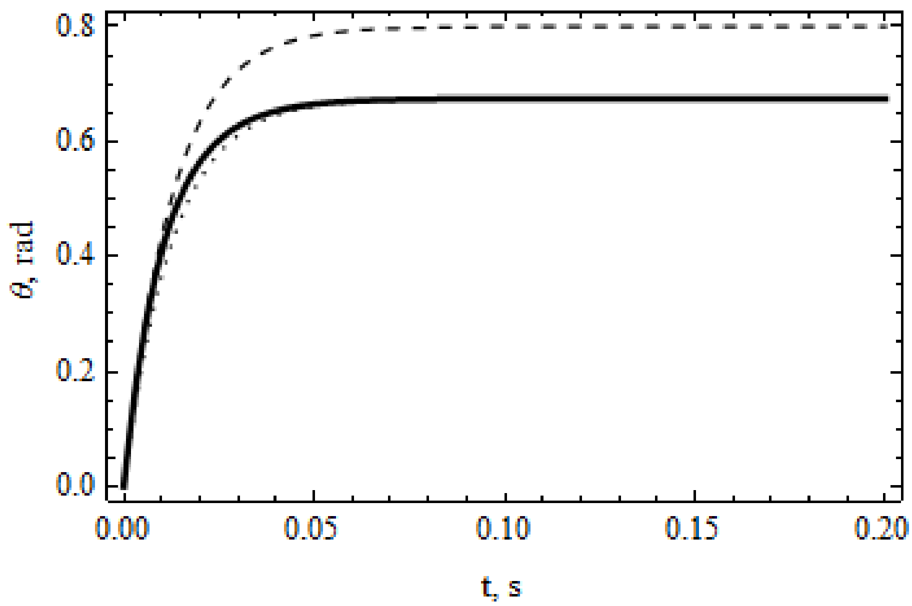

Fig. 3. Transient process simulations (solid – accurately simulated, dashed –

simplified approximation,dotted – improved approximation)

Improved transient process representation

Analysing expression (8) one can see than in steady state (s = 0) value of the θ angle is given by the simple ratio $g_2/2\zeta$. From the numerical simulation in Fig. 3 (dashed line) it is apparent that this value is not sufficiently accurate, while dynamic part appears to be acceptable. More accurate steady state value can be obtained directly from the expression (6), which results in the following improved approximation:

$$\theta(s) = \frac{1}{2}\mathrm{tg}^{-1}\left[\frac{4g_2\zeta_2\delta k}{\sqrt{\delta k^4 - 2(1-2\zeta_2^2)\delta k^2 + 1}}\right]\frac{\zeta_2 k\delta k}{s + \zeta_2 k\delta k}\delta\Omega(s). \qquad (10)$$

Numerical simulations of the improved approximation (10) are represented by the dotted line in Fig. 3. One can see that improved approximation (10) is accurate in representation of the angle of trajectory rotation transient processes for most of the practical applications.

Conclusions

Developed model for the angle of trajectory rotation of a CVG sensitive element allow designing miniature angular rate sensors based on the trajectory analysis contrary to the conventional secondary amplitude detection. Derived transfer functions can be used to develop filtering and control systems that will improve its measurement performances. The latter is suggested as a topic for the future research.

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