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TIME CHARACTERISTICS OF MICROSATELLITE ATTITUDE CONTROL AT INTERVALS BETWEEN REACTION WHEELS DESATURATIONS

Ua Стаття присвячена аналізу процесів зміни кутових швидкостей двигунів-маховиків мікросупутника під час його стабілізації, розвороту у надир та заспокоєння. Наведено вирази, що описують зміну кутових швидкостей двигунів-маховиків під час стабілізації супутника при дії постійних та гармонійних моментів на частоті, що дорівнює орбітальній кутовій швидкості. Досліджено вплив магнітного моменту, викликаного постійною складовою дипольного моменту супутника, на накопичення кінетичного моменту двигунів-маховиків. Формула для визначення амплітуди коливань кутових швидкостей двигунів-маховиків отримана з використанням моделі магнітного поля Землі у вигляді прямого диполя та кругової орбіти. При цьому її застосування для розрахунку максимальних значень кутових швидкостей дає результати дуже близькі до результатів моделювання з використанням точних моделей магнітного поля Землі і траєкторії руху центру мас супутника. Розглянуто процеси заспокоєння та розвороту мікросупутника за допомогою двигунів-маховиків. Оцінено максимально можливі інтервали часу роботи системи керування без розвантаження її виконавчих органів.

En The article is devoted to the processes study of changing the microsatellite reaction wheels angular velocities during its stabilization, turn and detumbling. Satellite control is considered under conditions of constant and harmonic external torques, as well as under gravitational, aerodynamic and magnetic disturbing torques. Expressions that describe the change in the reaction wheels angular velocities during satellite stabilization under the action of constant and harmonic torques at a frequency equal to the orbital angular velocity are given. The influence of the magnetic disturbing torque caused by the constant component of the satellite's dipole moment on the accumulation of the reaction wheels angular momentum has been studied. The formula for determining the magnitude of the reaction wheels angular velocities oscillations was obtained using a model of the Earth's magnetic field in the form of the straight dipole and taking into account a circular orbit. Moreover, its use for calculating the maximum values of angular velocities gives results that are very close to the results of modeling taking into account accurate models of the Earth’s magnetic field and the satellite’s center of mass trajectory. The maximum possible operating time intervals of the control system without forced desaturation of its controls are estimated.

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Introduction

The purpose of most modern artificial Earth satellites requires control of their rotational motion with a certain accuracy [1 – 5]. However, the motion of the satellite during some required control modes does not meet the demanded conditions for performing the main mission. Such modes are performed to prepare for it, and their create interruptions in the satellite's intended operation. These include the angular velocity damping mode, turning at a given angle, and reaction wheels desaturation. Developers of satellite control systems have to take into account the quality and time frames of their conducting, which affect mission planning [6 – 8].

Formulation of the problem

As a case study, this paper analyzes the time characteristics of the microsatellite rotational motion control processes in the modes of detumbling, turning at a given angle and stabilizing a given position without reaching the saturation of the reaction wheels angular velocities. At that, the main attention is paid to the accumulation process of angular momentum by reaction wheels.

Control object

The artificial Earth satellite in question moves in a sun-synchronous orbit at an altitude of 600 km with an inclination of -97.3 º. The microsatellite is equipped with four deployable solar panels. Satellite inertia tensor

$$I = \begin{pmatrix} 0{,}608 & 0{,}0082 & 0{,}039 \\ 0{,}0082 & 0{,}622 & 0{,}0044 \\ 0{,}039 & 0{,}0044 & 0{,}595 \end{pmatrix} \text{kg} \cdot \text{m}^2.$$

Reaction wheels with an axial moment of inertia of $1{,}6 \cdot 10^{-4}$ $\text{kg} \cdot \text{m}^2$ and a maximum rotation speed of 1200 rad/s are used as controls.

This paper studies the motion of the microsatellite by integrating nonlinear equations that describe the dynamics of it rotational motion and the center of mass motion dynamics in the field of magnetic, inertia, gravity, and aerodynamic forces.

Attitude stabilisation mode

Angular position stabilization is an automatic control mode during which the main task of the satellite in question is carried out. Its implementation is carried out using proportional-derivative control laws [9]:

$$
\begin{aligned}
M_{ux} &= K_x^{\gamma}\Delta\gamma + K_x^{\psi}\Delta\psi + K_x^{\vartheta}\Delta\vartheta + K_x^{\omega x}\Delta\omega_x + K_x^{\omega y}\Delta\omega_y + K_x^{\omega z}\Delta\omega_z - M_{gx};\\
M_{uy} &= K_y^{\gamma}\Delta\gamma + K_y^{\psi}\Delta\psi + K_y^{\vartheta}\Delta\vartheta + K_y^{\omega x}\Delta\omega_x + K_y^{\omega y}\Delta\omega_y + K_y^{\omega z}\Delta\omega_z - M_{gy};\\
M_{uz} &= K_z^{\gamma}\Delta\gamma + K_z^{\psi}\Delta\psi + K_z^{\vartheta}\Delta\vartheta + K_z^{\omega x}\Delta\omega_x + K_z^{\omega y}\Delta\omega_y + K_z^{\omega z}\Delta\omega_z - M_{gz},
\end{aligned}
\tag{1}
$$

where M_{ux}, M_{uy}, M_{uz} are control torques of reaction wheels; $\Delta\gamma, \Delta\psi, \Delta\vartheta$ are deviations of roll, yaw, pitch angles from specified values; $\Delta\omega_x, \Delta\omega_y, \Delta\omega_z$ are deviations of the satellite angular velocity projections on the body axes from the specified values; M_{gx}, M_{gy}, M_{gz} are the reaction wheels gyroscopic torque projections on the body axes; K_i^j are constant coefficients.

We will assume that in a steady state mode of operation, control torques (1) completely compensate disturbances. Then

$$
\begin{aligned}
I_m\dot{\omega}_{mx} &= I_m(\omega_{my}\omega_z - \omega_{mz}\omega_y) + M_{Bx};\\
I_m\dot{\omega}_{my} &= I_m(\omega_{mz}\omega_x - \omega_{mx}\omega_z) + M_{By};\\
I_m\dot{\omega}_{mz} &= I_m(\omega_{mx}\omega_y - \omega_{my}\omega_x) + M_{Bz},
\end{aligned}
\tag{2}
$$

where I_m is reaction wheel axial moment of inertia; $\omega_x, \omega_y, \omega_z$ are the projections of the satellite angular velocity on the body axes; $\omega_{mx}, \omega_{my}, \omega_{mz}$ are the angular velocities of the reaction wheels' own rotation; M_{Bx}, M_{By}, M_{Bz} are projections of the external forces disturbing torque acting on the satellite on the body axes.

In the absence of control errors, the body axes coordinate system coincide with the axes of the orbital coordinate system. Then the projections ω_x, ω_y will be harmonic functions with zero mean value and amplitude determined by the angular velocity of the Earth. If we neglect them, as well as the ellipticity of the orbit, system (2) will split into two independent parts with constant coefficients:

$$
\begin{aligned}
I_m\dot{\omega}_{mx} &= I_m\omega_{my}\omega_z + M_{Bx};\\
I_m\dot{\omega}_{my} &= -I_m\omega_{mx}\omega_z + M_{By};
\end{aligned}
\tag{3}
$$

and

$$I_m\dot{\omega}_{mz} = M_{Bz}. \tag{4}$$

At acting of constant disturbances

$$\omega_{mx} = (\omega_{mx0} + \frac{M_{Bx}}{I_m})\cos(\omega_z t) + (\omega_{my0} + \frac{M_{By}}{I_m})\sin(\omega_z t);$$
$$\omega_{my} = (\omega_{my0} + \frac{M_{By}}{I_m})\cos(\omega_z t) - (\omega_{mx0} + \frac{M_{Bx}}{I_m})\sin(\omega_z t); \quad (5)$$
$$\omega_{mz} = \omega_{mz0} + \frac{M_{Bz} t}{I_m},$$

where $\omega_{mx0}, \omega_{my0}, \omega_{mz0}$ are initial values of reaction wheels angular velocities accumulated in the transient operating mode of the control system; t is a time. Thus, at constant torques, an increase in angular velocity over time occurs only for the pitching reaction wheel [6].

However, simulating the system operation under the influence of real disturbing torques unlike (5) indicates an increase in the oscillations amplitude of angular velocities ω_{mx}, ω_{my} (fig. 1). It is caused by periodic disturbances at a frequency $-\omega_z$. The simulation results shown in fig. 1, fig. 2, obtained taking into account the aerodynamic, gravitational torques and the satellite dipole moment of $\{1,\ 3,\ 5\} \cdot 10^{-2} A \cdot m^2$.

Let us consider the behavior of system (3) under harmonic disturbance:

$$M_{Bx} = M_{xc}\cos(\omega_z t) + M_{xs}\sin(\omega_z t);$$
$$M_{By} = M_{yc}\cos(\omega_z t) + M_{ys}\sin(\omega_z t). \quad (6)$$

Indeed, the decision of it

$$\omega_{mx} = \left(\omega_{mx0} + \frac{M_{xc} - M_{ys}}{2I_m} t\right)\cos(\omega_z t) +$$
$$+ \left(\omega_{my0} + \frac{M_{xc} + M_{ys}}{2\omega_z I_m} + \frac{M_{xs} + M_{yc}}{2I_m} t\right)\sin(\omega_z t);$$
$$\omega_{my} = \left(\omega_{my0} + \frac{M_{xs} + M_{yc}}{2I_m} t\right)\cos(\omega_z t) + \quad (7)$$
$$+ \left(\frac{M_{yc} - M_{xs}}{2\omega_z I_m} - \omega_{mx0} + \frac{M_{ys} - M_{xc}}{2I_m} t\right)\sin(\omega_z t);$$

contains harmonics with amplitude increasing in proportion to time.

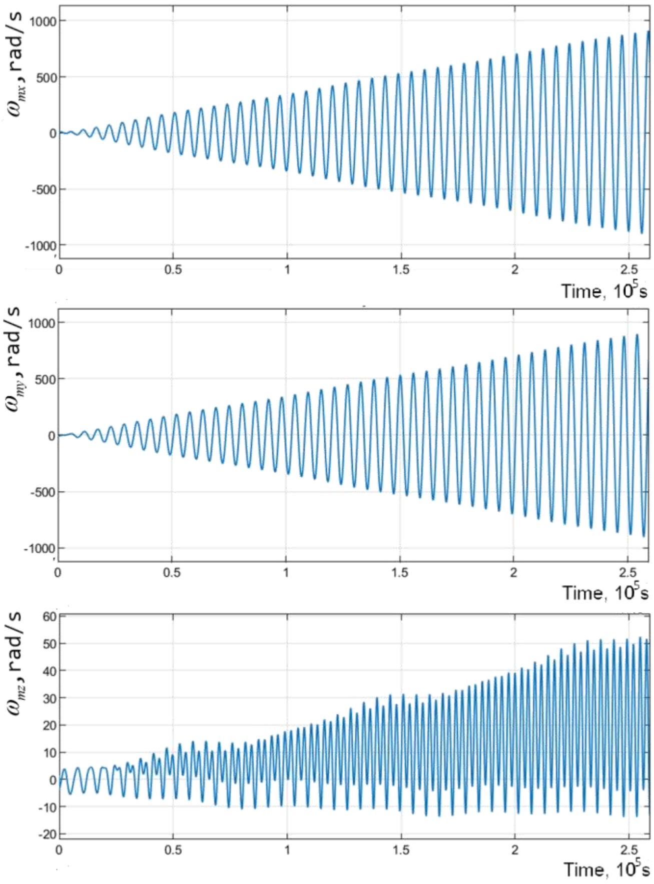

Fig. 1. Angular velocities of reaction wheels in stabilization mode under the action of aerodynamic, gravitational and magnetic torques

Thus, at an arbitrary moment in time, after t seconds since the end of the transient process in the control system, the amplitudes of the reaction wheels angular velocities oscillations

$$\omega_{mx\max}=\sqrt{\left(\omega_{mx0}+\frac{M_{xc}-M_{ys}}{2I_m}t\right)^2+\left(\omega_{my0}+\frac{M_{xc}+M_{ys}}{2I_m\omega_z}+\frac{M_{xs}+M_{yc}}{2I_m}t\right)^2};$$
$$\omega_{my\max}=\sqrt{\left(\omega_{my0}+\frac{M_{yc}+M_{xs}}{2I_m}t\right)^2+\left(\frac{M_{yc}-M_{xs}}{2I_m\omega_z}-\omega_{mx0}+\frac{M_{ys}-M_{xc}}{2I_m}t\right)^2}. \tag{8}$$

However, from (7) it is also clear that at zero initial velocities over a significant time interval, amplitudes (8) are approximately equal to each other and are mainly determined by components with an amplitude proportional to time:

$$\omega_{mxt}=\omega_{myt}=\frac{t}{2I_m}\sqrt{(M_{xc}-M_{ys})^2+(M_{xs}+M_{yc})^2}. \tag{9}$$

The components corresponding to expressions (6) are contained in magnetic torque. They arise in the presence of a constant part of the satellite's dipole moment. Let's consider its effect on the rotation speed of reaction wheels, taking into account the Earth's magnetic field in accordance with the direct dipole model.

When stabilizing the satellite along the axes of the orbital coordinate system, the projections of the magnetic torque

$$M_{mx} = (-m_y \cos i + 2m_z \sin i \sin u)\frac{\mu}{r^3};$$
$$M_{my} = (m_x \cos i + m_z \sin i \cos u)\frac{\mu}{r^3}; \qquad (10)$$
$$M_{mz} = (-2m_x \sin i \sin u - m_y \sin i \cos u)\frac{\mu}{r^3},$$

where m_x, m_y, m_z are projections of the satellite dipole moment on the body axes; i is orbit inclination; u is satellite latitude argument; $\mu = 8\cdot 10^{15} T \cdot m^3$; r is distance from the satellite to the center of the Earth.

In expressions (10) the terms proportional to $m_z \sin i$ changes with frequency $-\omega_z$. They are the ones that can cause in solution (7) of system (3) a proportional to time increase in the oscillations amplitude. To reduce this effect, when preparing satellites for flight, as well as during their development, it is necessary to strive to reduce the constant component of m_z. Obviously, the described phenomenon is especially noticeable for circumpolar orbits.

Comparing (6) and (10), we can determine the corresponding amplitudes of the magnetic torque:

$$M_{xc} = 0;\ M_{xs} = -2\frac{\mu}{r^3} m_z \sin i;$$
$$M_{ys} = 0;\ M_{yc} = \frac{\mu}{r^3} m_z \sin i. \qquad (11)$$

As a result of substituting (11) into (9), we will have an expression for the component of the reaction wheels angular velocities ω_{mx}, ω_{my} amplitude, which depends on time:

$$\omega_{mxt} = \omega_{myt} = \frac{m_z \mu \sin i}{2 I_m r^3} t. \qquad (12)$$

At zero initial angular velocities $\omega_{mx0}, \omega_{my0}$ and a sufficiently long operating time, expression (12) is approximately equal to the entire amplitude of oscillations ω_{mx}, ω_{my}, and it can be used to estimate the maximum possible speeds of the reaction wheels due to the action of the satellite's constant dipole moment.

According to (12), the amplitudes of angular velocities for the conditions of the simulation experiment, the results of which are shown in fig. 1 – fig. 3, after three days they amounted to 945 rad/s, that is, more than $3/4$ of the maximum possible. The closeness of the results obtained during simulation to the calculated ones (Table 1) indicates the validity of the assumptions and simplifications adopted in the analysis of the reaction wheels motion dynamics. Their difference is largely due to the use of a direct dipole model to obtain (12), and in the simulation experiment, an accurate model of the Earth's magnetic field (12^{th} order) [10].

***Table* 1.**

	maximums		minimums		maximums		minimums	
	ω_{mx}, rad/s	ω_{mx}, rad/s	ω_{mx}, rad/s	ω_{mx}, rad/s	ω_{my}, rad/s	ω_{my}, rad/s	ω_{my}, rad/s	ω_{my}, rad/s
Time, s	258400	142946	255518	140064	254062	144370	256946	141484
Simulation result	916,85	516,97	–906,43	–509,04	903,38	521,66	–910,95	–511,4
Amplitude according to (12)	942,88	521,6	932,36	511,08	927,0510	526,79	937,57	516,26

The nature of the change in angular velocity ω_{mz} of pitch reaction wheel (fig. 2) differs significantly from the simplified model (4). Solution of (4) in the presence of simulation experiment disturbing torques

$$\omega_{mz} = \omega_{mz0} + \frac{M_{Bz0}}{I_m} t - \frac{\mu \sin i}{I_m \omega_z r^3} \sqrt{4m_x^2 + m_y^2} \sin\left(\omega_z t + \operatorname{arctg} \frac{2m_x}{m_y}\right), \qquad (13)$$

contains a trend with a slope of $1{,}1 \cdot 10^{-4}$ rad/s^2 and a harmonic with a constant amplitude of 4,9 rad/s. In (13) M_{Bz0} is the sum of the projections of the aerodynamic and gravitational torques on the axis z. However, in fig. 1 there is an increase in amplitude ω_{mz}, caused by stabilization errors, that is, inaccurate coincidence of the body axes and orbital coordinate system axes. The errors are periodic in nature with maximum values: of $8 \cdot 10^{-4}$ rad for the roll angle, $1{,}9 \cdot 10^{-3}$ rad for yaw angle, and $4 \cdot 10^{-4}$ rad for pitch angle (fig. 2).

The behavior of the pitch reaction wheel, analytically described by expression (13), can be illustrated using a simulation experiment in which, together with the aerodynamic and gravitational torques, there is a magnetic torque caused by a satellite constant dipole moment with zero projection along the z axis: $\{1,\ 3,\ 0\} \cdot 10^{-2} A \cdot m^2$ (fig. 3).

In this case, according to (7), (12), the projections of the reaction wheels angular momentum $I_m\omega_{mx}$, $I_m\omega_{my}$ on the axis of the orbital coordinate system do not contain harmonics with increasing amplitude. Therefore, in the presence of stabilization errors, there are no such components in its projection onto the z axis of the body coordinate system (fig. 3). On the other hand, the absence of an increase in amplitude of ω_{mx}, ω_{my} in fig. 3 indicates that the source of this increase in the previous experiment (fig. 1) is indeed the projection m_z of the satellite dipole moment.

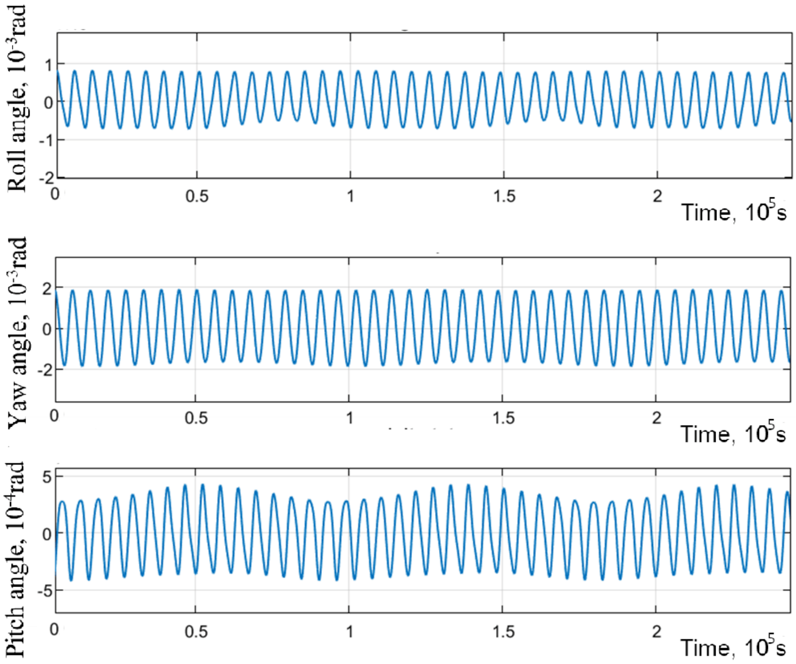

Fig. 2. Satellite stabilization errors

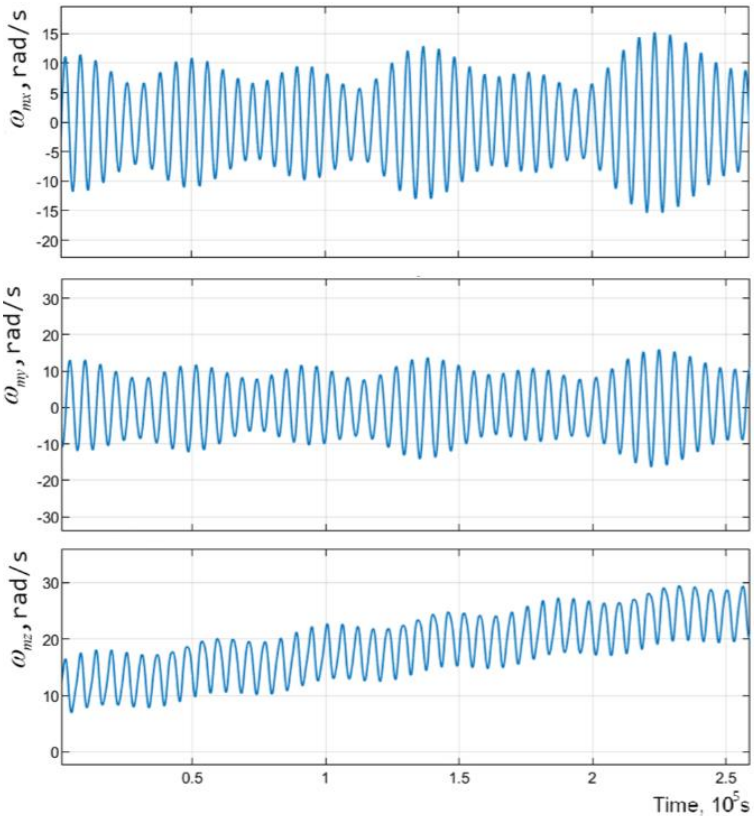

Fig. 3. Angular velocities of reaction wheels in stabilization mode in the absence of satellite dipole moment projection m_z

Control mode of satellite turn

The turn of the microsatellite under study is also carried out using the control law (1). Since its main task is performed when the body axes coincide with the axes of the orbital coordinate system, we consider turning to this position from an arbitrary initial one (fig. 4, fig. 5). In this case, the nature of the transient processes for different initial orientation angles is different, and the duration does not exceed 100 s (fig. 4).

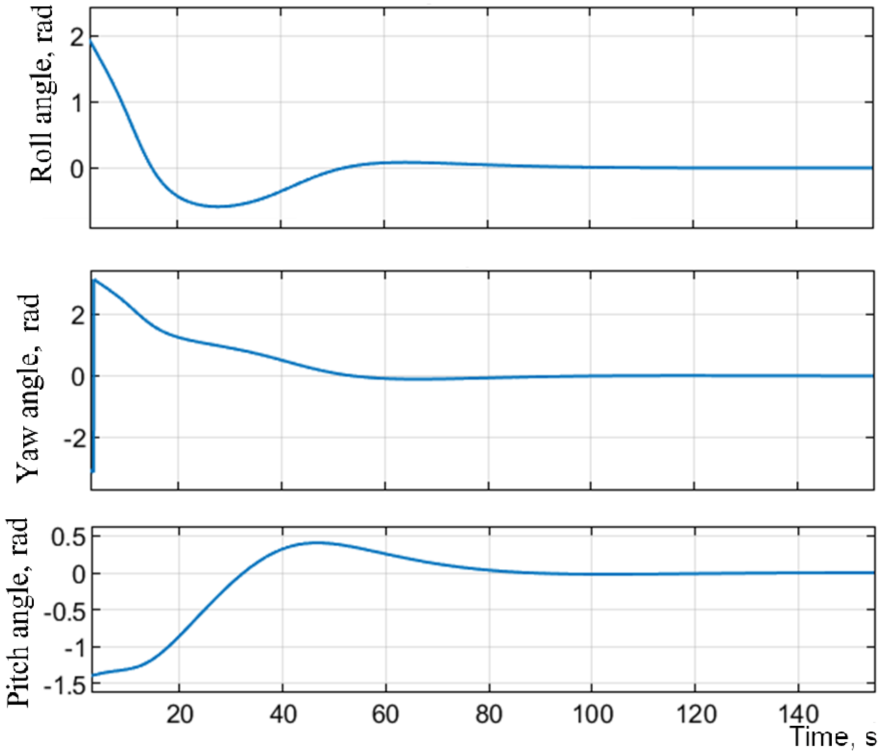

Fig. 4. Satellite turn to the position of the orbital coordinate system

The angular speeds of the reaction wheels during a turn reach 250 rad/s. However, after the end of the transition process, the flywheels slow down: the pitching one almost to a complete stop, and the angular velocity of the others does not exceed 10 rad/s (fig. 5).

Simulation, the results of which are shown in fig. 4, fig. 5, was carried out for zero initial angular velocities of reaction wheels. If the flywheels rotate quickly when the turn begins, a significant angular momentum remains after it ends.

Detumbling the satellite using reaction wheels

In the mode of satellite calming using reaction wheels, a proportional control laws are used:

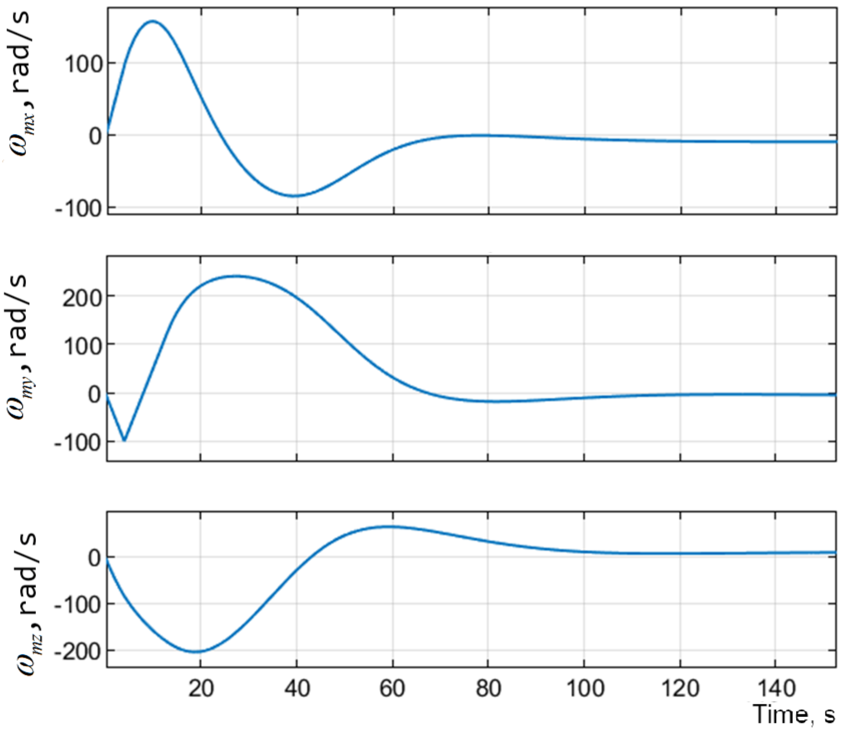

Fig. 5. Angular velocities of reaction wheels when turning the satellite

$$\begin{aligned} M_{ux} &= K_x^{\omega x}\Delta\omega_x + K_x^{\omega y}\Delta\omega_y + K_x^{\omega z}\Delta\omega_z - M_{gx}; \\ M_{uy} &= K_y^{\omega x}\Delta\omega_x + K_y^{\omega y}\Delta\omega_y + K_y^{\omega z}\Delta\omega_z - M_{gy}; \\ M_{uz} &= K_z^{\omega x}\Delta\omega_x + K_z^{\omega y}\Delta\omega_y + K_z^{\omega z}\Delta\omega_z - M_{gz}. \end{aligned} \tag{14}$$

They allows to stop the satellite at an arbitrary ratio of the projections of its initial angular velocity in a time not exceeding 20 minutes (fig. 6, fig. 7).

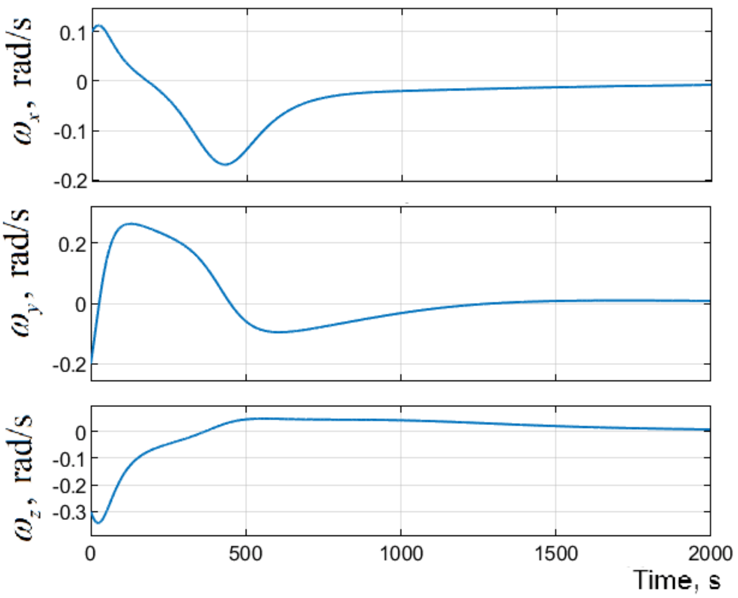

Fig. 6. Angular velocity of the satellite during the detumbling process

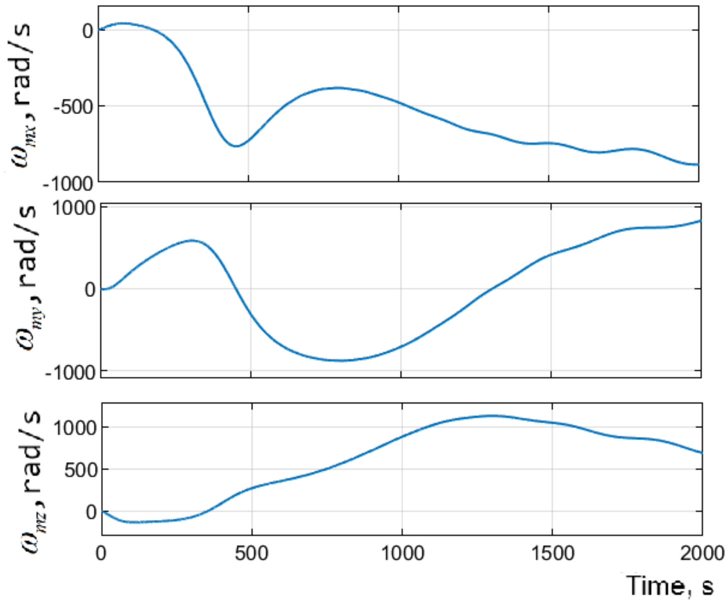

Fig. 7. Angular velocities of reaction wheels in the satellite detumbling process

In this case, there is a large margin of control torque, and if necessary, this time can be significantly reduced by increasing the coefficients K_i^j of control laws (14). However, the absolute value of the initial angular velocity, at which a complete stop is possible without desaturation the reaction wheels, is limited by their maximum angular momentum. For the satellite in question, it should be less than 0,4 rad/s. During the satellite's detumbling with an initial angular velocity of 0,37 rad/s, the angular velocities of the reaction wheels approach saturation.

Unlike the turn mode, after the completion of the satellite stopping process, the reaction wheels do not slow down (fig. 7).

Conclusions

The study of the microsatellite behavior in the modes of detumbling, turning at a given angle and the attitude stabilisation revealed features of the time characteristics of control processes that affect the time intervals for performing the main task and preparing for it without reaching the reaction wheels saturation.

When the satellite is stabilized in the orbital coordinate system, the angular velocities ω_{mx}, ω_{my} of its reaction wheels have harmonic components with an amplitude that increases in proportion to time. They are caused by a harmonic disturbing torque at a frequency equal to the orbital angular velocity of the satellite. The resulting formulae make it possible to calculate this amplitude with known parameters of the disturbing torque.

The main disturbance for the satellite in question at this frequency is the torque caused by the constant component of its dipole moment. Analysis of its influence on the reaction wheels angular velocities indicates the determining role in the increase in their amplitude of the dipole moment projection on the axis perpendicular to the orbital plane. This influence is most pronounced in circumpolar orbits.

Oscillation amplitude calculation results according to the given formula are very close to the values obtained in the simulation experiment. They show that during three days of stabilization the angular velocities ω_{mx}, ω_{my} exceeded ¾ of their maximum values. In a similar simulation experiment, but with a zero projection m_z of the dipole moment during the same time they reached only 15 rad/s.

The angular velocity of the pitch reaction wheel obtained from the simulation, in addition to the expected trend and harmonic function with a constant amplitude, has a periodic function with increasing amplitude. Maximum value of ω_{mz} over three days of stabilization was 52 rad/s. The increase in the amplitude of oscillations arose due to stabilization errors. However, when $m_z = 0$ periodic functions with increasing amplitude are not noticeable. In this case, in three days the pitch reaction wheel spun up to only 30 rad/s.

Thus, known measures to eliminate the dipole moment of the satellite in question, applied to a projection perpendicular to the orbit, can increase the time of its continuous operation without desaturation the reaction wheels from the existing four to one hundred days.

During detumbling the satellite in question, the reaction wheels quickly accumulate angular momentum. After this mode, unloading should be carried out, even if the speeds of the reaction wheels are far from saturation. It will ensure their almost complete stop at the end of the subsequent satellite turning.

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