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FREQUENCY CHARACTERISTIC OF A UNIFORMLY ROTATING LASER GYRO IN EXACT AND POLYNOMIAL FORMS

Ua У деяких інерціальних навігаційних системах карусельного типу лазерні гіроскопи працюють в режимі рівномірного обертання. Для розробки таких систем та комп'ютерного моделювання їх роботи необхідно мати «точний» (у широкому діапазоні кутових швидкостей Ω) аналітичний вираз для частоти биття $\omega_{beat} = \omega_{beat}(\Omega)$ зустрічних електромагнітних хвиль рівномірно обертового приладу. Цей вираз може бути отриманий шляхом розв'язання широко відомої системи динамічних рівнянь лазерного гіроскопа з точністю до другого порядку по параметрах лінійного зв'язку зустрічних хвиль. Однак перед тим, як використовувати такі гіроскопи у складі вказаної інерціальної системи, – їх метрологічні параметри (кутові ціни імпульсу та зміщення нуля) мають бути попередньо відкалібровані. Для синтезу методики такої калі-

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бровки на швидко обертовій платформі одновісного стенда та відповідного аналізу її методичних похибок, – необхідно також мати наближений аналітичний вираз для частоти биття зустрічних хвиль лазерного гіроскопа у формі полінома

$$\omega_{beat}^{P} = K_{(1)}\,\Omega + K_{(0)} + K_{(-1)}\,\Omega^{-1} + K_{(-2)}\,\Omega^{-2} + K_{(-3)}\,\Omega^{-3} + K_{(-4)}\,\Omega^{-4}$$

з коефіцієнтами $K_{(1)}$, ..., $K_{(-4)}$, котрі містять у собі параметри точного виразу для ω_{beat}. Як показує аналіз літератури, відомий вираз для ω_{beat} та відомі співвідношення для деяких з коефіцієнтів полінома ω_{beat}^{P} є неповними і тому мають бути відкориговані. У статті представлено результат такого коригування.

En In some inertial navigation systems of a carousel type, laser gyros operate in regime of uniform rotation. For development of such systems and computer simulation of their work, one needs to have the “exact” (in wide range of angular velocities Ω) analytical expression for counterpropagating waves beat frequency $\omega_{beat} = \omega_{beat}(\Omega)$ of uniformly rotating device. This expression may be obtained by solving the well-known system of laser gyro dynamic equations with accuracy to second order in parameters of counterpropagating waves linear coupling. But before use of such non-dithered laser gyros in the named inertial system, their metrological parameters (scale factors and null shifts) must be preliminary calibrated. For synthesis and qualitative analysis of methodical errors of the procedure of such calibration on a quickly rotating platform of a single-axis stand, one needs also to have the approximate analytical expression for counterpropagating waves beat frequency in the form of polynomial

$$\omega_{beat}^{P} = K_{(1)}\,\Omega + K_{(0)} + K_{(-1)}\,\Omega^{-1} + K_{(-2)}\,\Omega^{-2} + K_{(-3)}\,\Omega^{-3} + K_{(-4)}\,\Omega^{-4}$$

with coefficients $K_{(1)}$, ..., $K_{(-4)}$ which involve parameters of the exact expression for ω_{beat}. As analysis of the literature shows, the known expression for ω_{beat} and the known relations for some of coefficients of polynomial ω_{beat}^{P} are not complete and, therefore, must be modified. In the paper, the result of such modification is presented.

Introduction

Among the main types of laser gyros that are widely used in practice, one can highlight a device on a base of a ring $He-Ne$ gas laser ($^{20}Ne : {}^{22}Ne = 1{:}1$) with a planar N-mirror ($N = 3,\ 4$) resonator which provides radiation linearly polarized in the sagittal plane. Pumping of the laser which operates, as a rule, at wavelength $\lambda_0 = 0.6328\times10^{-6}$ m, is realized by means of a constant discharge current with use a symmetric scheme: one cathode – two anodes [1] – [5].

According to relations (6.45) – (6.47) from [6], the system of equations describing the dynamics of dimensionless intensities I_j ($j = 1,\ 2$) and phase difference ψ of counterpropagating waves of such laser gyro (under condition of equal currents in its discharge legs) may be presented in the form

$$\dot{I}_1 = (\alpha_1 - \beta I_1 - \theta I_2) I_1 - 2 r_2 (I_1 I_2)^{1/2} \cos(\psi + \varepsilon_2),$$
$$\dot{I}_2 = (\alpha_2 - \beta I_2 - \theta I_1) I_2 - 2 r_1 (I_1 I_2)^{1/2} \cos(\psi - \varepsilon_1), \tag{1}$$
$$\dot{\psi} = M\Omega + (\rho - \tau)(I_1 - I_2) + r_2 (I_2/I_1)^{1/2} \sin(\psi + \varepsilon_2) + r_1 (I_1/I_2)^{1/2} \sin(\psi - \varepsilon_1).$$

In deriving these equations it was taken into account that the electromagnetic wave with $j = 1$ propagates in the direction of the laser gyro rotation.

In system (1): α_j, β, θ, ρ, τ are the Lamb coefficients which characterize the properties of the active medium; $M = (1 + K_a) M_g$ is the laser gyro scale multiplier which is determined mainly by its geometrical component $M_g = 8\pi A/(\lambda_0 L)$ (L – axis contour perimeter, A – area enclosed by axis contour), but which takes into account also the properties of the active medium ([7]) by means of very small parameter K_a ($K_a < 0$, $|K_a| <<< 1$); Ω is the angular velocity with which the laser gyro rotates in the inertial space; r_j and ε_j are the modules and arguments of complex integral coefficients $r_j \exp\{\varepsilon_j\}$ of counterpropagating waves linear coupling, characterizing their interaction through backscattering, absorption, and transmission of radiation on the mirrors.

In some inertial navigation systems of a carousel type (see paragraphs 3.7.5 and 3.8.7 in [5]), laser gyros operate in regime of uniform rotation. They are mounted on a special platform which continuously, during each cycle, performs n full revolutions in ccw-direction, and then n full revolutions in cw-direction. For development of such systems and computer simulation of their work, one needs to have the “exact” (in wide range of Ω) analytical expression for counterpropagating electromagnetic waves beat frequency ω_{beat} of uniformly rotating device. Such expression may be obtained as a result of solving system (1) with accuracy to second order in parameters r_j of counterpropagating waves linear coupling. If relation $\omega_{beat} = \omega_{beat}(\Omega)$ is known, then the number of information pulses ΔN, accumulated on the laser gyro output during time Δt, can be found from the differential equation

$$\frac{dN}{dt} = \frac{k_f}{2\pi} \omega_{beat}, \tag{2}$$

where dN/dt is the pulse repetition rate on the gyro output, and k_f is the “frequency multiplication coefficient” (as a rule, $k_f = 1, 2, 4$). In the literature, equation (2) is called “frequency characteristic of a uniformly rotating laser gyro”.

But before use of such laser gyros in the named inertial system, their metrological parameters (scale factors and null shifts) must be preliminary calibrated (see item *b* of point 12.9.3.1 in [8]). For synthesis of the procedure of such calibration on a quickly rotating platform of a single-axis stand, and corresponding analysis of its methodical errors, – one needs also to have the approximate analytical expression for counterpropagating waves beat frequency in asymptotic limit of high values of Ω. This relation may be taken in the form of polynomial

$$\omega_{beat}^{P} = K_{(1)}\Omega + K_{(0)} + K_{(-1)}\Omega^{-1} + K_{(-2)}\Omega^{-2} + K_{(-3)}\Omega^{-3} + K_{(-4)}\Omega^{-4} \qquad (3)$$

with coefficients $K_{(1)}$, ..., $K_{(-4)}$ which involve parameters of the exact expression for ω_{beat}.

Results of analysis of the literature

Known exact expression for ω_{beat}

As analysis of the literature shows, the known expression for ω_{beat} (calculated on the base of system (1) to *second* order in parameters r_j of counterpropagating waves linear coupling) has the form

$$\begin{aligned} \omega_{beat} = \omega_0 &+ [1 - \frac{\omega_{s(0)}^2}{2\omega^2} + \frac{(1+4T^2)r_m^2}{2(\alpha_m^2+\omega^2)}]\omega + \\ &+ D(r_2^2 - r_1^2)\{-\frac{1}{\omega^2} + \frac{1}{\alpha_m^2+\omega^2}[1 + \frac{\alpha_m(\alpha_p+\alpha_m)}{2(\alpha_p^2+\omega^2)}] - \\ &- \frac{2T^2\alpha_m^2}{\omega^2(\alpha_m^2+\omega^2)}[1 + \frac{\alpha_p}{\alpha_m}\frac{\alpha_p\alpha_m - \omega^2}{\alpha_p^2+\omega^2}]\}\omega. \end{aligned} \qquad (4)$$

The first term in the right-hand side of (4) is well-known (see, for example, [1, 6, 9,[10]). The second term is known from [10] (see formulas (6.31), (6.32), (6.7) therein). And the third term is known from [11] (see expression (23) therein).

As one can see, the second and the third terms in (4) describe only the *reversible* (with respect to Ω) components of ω_{beat}. There are not the *nonreversible* ones in (4).

NOTE: In the literature, in works [11, 12, 13], in addition to (4), there are correspondingly three qualitatively different versions of expressions for the nonreversible components of ω_{beat}. It is important to note that these relations are obtained with accuracy to the *fourth* order in parameters r_j. Their common feature is that they are proportional to the combination $(r_2^2 - r_1^2)r_1 r_2 \sin\varepsilon_{12}$ which describes the influence of the factor of asymmetry ($r_1 \neq r_2$) of counterpropagating waves linear coupling. As it will be shown below, such factor is less significant than another one – the factor of inequality ($\alpha_1 \neq \alpha_2$) of counterpropagating waves amplification due to nonreciprocal resonator losses. This more important factor will manifest itself already to *second* order in parameters r_j.

In (4), the following notations are used:

- $\omega = M\Omega$ is the laser gyro counterpropagating waves frequencies splitting caused by its rotation in the inertial space with angular velocity Ω (in ideal device $\omega_{beat} = \omega = M\Omega$);
- $\varepsilon_{12} = \varepsilon_1 + \varepsilon_2$ is the sum of arguments of complex integral coefficients $r_j \exp\{\varepsilon_j\}$ of counterpropagating waves linear coupling;
- $r_p = (r_1^2 + r_2^2 + 2r_1 r_2 \cos\varepsilon_{12})^{1/2}$ and $r_m = (r_1^2 + r_2^2 - 2r_1 r_2 \cos\varepsilon_{12})^{1/2}$ are the combinations of modules and arguments of these coefficients;
- $\alpha_p = \alpha = (1/2)(\alpha_1 + \alpha_2)$ is the inverse relaxation time of the sum of counterpropagating waves intensities;
- $\alpha_m = \alpha_p (1-h)/(1+h)$ is the inverse relaxation time of the difference of counterpropagating waves intensities. Here $h = \theta/\beta$ is the dimensionless parameter which depends linearly on the $He-Ne$ mixture total pressure;
- $D = (1/2)\alpha_m^{-1}(\alpha_2 - \alpha_1)$ is the small dimensionless parameter which characterizes the degree of inequality of counterpropagating waves amplification caused by difference of the laser gyro resonator losses. In ideal device, the losses for both waves are equal, and $D = 0$;
- $T = (\rho - \tau)/(\beta - \theta)$ is the small dimensionless parameter which characterizes the degree of laser gyro resonator frequency detuning from the center of the active medium emission line. This detuning is caused by a small (with respect to λ_0) systematic error ΔL of the perimeter control extremum system (its periodical search steps are not considered). In ideal case of accurate laser gyro resonator frequency tuning to the line center, $\Delta L = 0$, $\rho = \tau = 0$, and $T = 0$;
- $\omega_0 = -2TD\alpha_m = -T(\alpha_2 - \alpha_1)$ is the so-called “null shift” of the laser gyro frequency characteristic. In other words, it is the counterpropagating waves frequencies splitting (even when $\Omega = 0$) caused by a multiplicative interaction of the factor of unequal waves amplification and the factor of resonator frequency detuning. If quantity ω_0 is known, then the laser gyro null shift Ω_0, which has dimension of angular velocity, may be calculated as $\Omega_0 = \omega_0/M$;
- $\omega_{s(0)} = (r_p^2 + 8T r_1 r_2 \sin\varepsilon_{12} + 4T^2 r_m^2)^{1/2}$ is the parameter which has dimension of angular frequency and characterizes the halfwidth of synchronization

zone of the laser gyro counterpropagating waves frequencies in approximation $D = 0$. If quantity $\omega_{s(0)}$ is known, then the laser gyro synchronization zone halfwidth $\Omega_{s(0)}$, which has dimension of angular velocity, may be calculated in such approximation as $\Omega_{s(0)} = \omega_{s(0)} / M$.

NOTE: In the case $D = T = 0$, expression (4) has the form $\omega_{beat} = \{1 - r_p^2/(2\omega^2) + r_m^2/[2(\alpha_m^2 + \omega^2)]\}\omega$ which is confirmed theoretically and experimentally in [14].

Known expressions for coefficients of polynomial ω_{beat}^P

As analysis of the literature shows, the known relations for coefficients $K_{(1)}$, ..., $K_{(-4)}$ of polynomial ω_{beat}^P (calculated on the base of system (1) to *second* order in parameters r_j) have the following forms:

Coefficients $K_{(1)}$ and $K_{(0)}$. As it follows directly from (4),

$$K_{(1)} = M, \tag{5}$$

and, in accordance, for example, with [1], [6], [9], [10],

$$K_{(0)} = \omega_0, \quad \omega_0 = -T(\alpha_2 - \alpha_1), \quad T = (\rho - \tau)/(\beta - \theta). \tag{6}$$

Coefficients $K_{(-1)}$ and $K_{(-3)}$. According to [9], [10], [15]–[17],

$$K_{(-1)} = -2 r_1 r_2 (\cos\varepsilon_{12} + 2T \sin\varepsilon_{12}) / M, \tag{7}$$

and, in accordance with [17],

$$K_{(-3)} = -(1/2)(1 + 4T^2)\alpha_m^2 r_m^2 / M^3. \tag{8}$$

Coefficients $K_{(-2)}$ and $K_{(-4)}$. In the literature, there are not yet expressions for these coefficients calculated to *second* order in parameters r_j. So the corresponding relations for these quantities must be found.

NOTE: In the literature ([11], [12], [13]), there are qualitatively different expressions for these coefficients, but calculated to the *fourth* order in parameters r_j. Their common feature is that they are proportional to the combination $(r_2^2 - r_1^2) r_1 r_2 \sin\varepsilon_{12}$ which describes the influence of the factor of asymmetry ($r_1 \neq r_2$) of counterpropagating waves linear coupling.

Principal goals of this paper

As analysis of the literature shows, the known exact expression (4) for counterpropagating waves beat frequency ω_{beat} and the known relation (8) for coefficient $K_{(-3)}$ of polynomial ω_{beat}^{P} are not complete: they do not reflect fully the influence of the factor of inequality ($\alpha_1 \neq \alpha_2$) of counterpropagating waves amplification due to nonreciprocal resonator losses. Moreover, the sought for (to *second* order in parameters r_j) expressions for coefficients $K_{(-2)}$ and $K_{(-4)}$ of ω_{beat}^{P} are still unknown. So the principal goals of this paper are: 1) to propose the modified exact expression for ω_{beat}; 2) to confirm the known relations for coefficients $K_{(1)}$, $K_{(0)}$, and $K_{(-1)}$ of polynomial ω_{beat}^{P}; 3) to present the modified expression for coefficient $K_{(-3)}$; 4) to find the sought for relations for coefficients $K_{(-2)}$ and $K_{(-4)}$.

Modified exact expression for ω_{beat}

According to the author's report [18], the modified exact expression for ω_{beat} may be written in the form

$$
\begin{aligned}
\omega_{beat} = {} & \omega_0 + \Big[1 - \frac{\omega_{s(0)}^2}{2\omega^2} + \frac{(1+4T^2)\,r_m^2}{2(\alpha_m^2+\omega^2)}\Big]\omega + \\
& + D(r_2^2 - r_1^2)\Big\{-\frac{1}{\omega^2} + \frac{1}{\alpha_m^2+\omega^2}\Big[1 + \frac{\alpha_m(\alpha_p+\alpha_m)}{2(\alpha_p^2+\omega^2)}\Big] - \\
& - \frac{2T^2\alpha_m^2}{\omega^2(\alpha_m^2+\omega^2)}\Big[1 + \frac{\alpha_p}{\alpha_m}\frac{\alpha_p\alpha_m-\omega^2}{\alpha_p^2+\omega^2}\Big]\Big\}\omega + \\
& + (1+4T^2)(\alpha_2-\alpha_1)\,r_1 r_2 \sin\varepsilon_{12}\,\frac{\alpha_p\alpha_m-\omega^2}{2(\alpha_p^2+\omega^2)(\alpha_m^2+\omega^2)} + \\
& + \omega_0\Big[\frac{\omega_{s(0)}^2}{2\omega^2} + (1+4T^2)\,r_m^2\,\frac{\alpha_m^2-\omega^2}{2(\alpha_m^2+\omega^2)^2}\Big].
\end{aligned}
\tag{9}
$$

Expression (9) is derived on the base of system (1) with the help of the procedure which is generalization (for the case $T \neq 0$) of the method of successive approximations developed earlier by the author in [19] for the case $T = 0$. The first, the second, and the third terms in (9) confirm the known expression (4) for ω_{beat}. But the fourth and the fifth terms in (9) are substantially new.

They describe two nonreversible with respect to Ω components of ω_{beat} caused by the factor of inequality of counterpropagating waves amplification due to nonreciprocal resonator losses.

Expression (9) may be used in the range $[\Omega \le -(3\Omega_s + \Omega_0)$; $\Omega \ge +(3\Omega_s - \Omega_0)]$, where $\Omega_0 = \omega_0 / M$, and

$$\Omega_s = [r_p^2 + 8T r_1 r_2 \sin \varepsilon_{12} + 4T^2 r_m^2 + 2D(r_2^2 - r_1^2)(1 + 4T^2)]^{1/2} / M \qquad (10)$$

is the laser gyro synchronization zone halfwidth calculated for general case $D \ne 0$.

Expression (9) is valid, if the condition of weakness of counterpropagating waves linear coupling is fulfilled. It implies that for all given possible values of laser gyro total discharge current, the values of ratios r_p / α_p and r_m / α_m must be much less than unity. In modern devices, operating with sufficiently high level of pumping (see paragraph 3.3.2 in [5]), the named condition, as a rule, is satisfied.

NOTE: For the laser gyro (with a four-mirror square resonator) operating at total pressures of the $He - Ne$ mixture from 1 to 5–6 Torr, a set of engineer formulas for calculation of the parameters α_j, β, θ, ρ, τ, K_a, and r_j, ε_j (case $r_1 = r_2 = r$, $\varepsilon_1 = \varepsilon_2 = \varepsilon$) of system (1) is proposed in [20]. A set of relations for estimating the parameters r_j, ε_j of such laser gyro for general case ($r_1 \ne r_2$, $\varepsilon_1 \ne \varepsilon_2$) is presented in [21]. Formulas for simulating the dynamics of the parameters r_j, ε_j during the device operation in the self-heating regime are proposed in [22].

Modified expressions for coefficients of polynomial ω_{beat}^P

The modified expressions for coefficients $K_{(1)}$, ..., $K_{(-4)}$ of polynomial ω_{beat}^P may be obtained on the base of relation (9) for ω_{beat} with the help of the following approximate formulas (which are valid for $|\omega| >> \alpha_p, \alpha_m$):

$$\begin{aligned}
F_1 &= \omega / (\alpha_m^2 + \omega^2) \approx 1/\omega - \alpha_m^2 / \omega^3, \\
F_2 &= 1/[\omega(\alpha_m^2 + \omega^2)] \approx 1/\omega^3, \\
F_3 &= 1/(\alpha_m^2 + \omega^2) \approx 1/\omega^2 - \alpha_m^2 / \omega^4, \\
F_4 &= \omega^2 / (\alpha_m^2 + \omega^2) \approx 1 - \alpha_m^2 / \omega^2 + \alpha_m^4 / \omega^4, \\
F_5 &= 1/(\alpha_m^2 + \omega^2)^2 \approx 1/\omega^4, \\
F_6 &= \omega^2 / (\alpha_m^2 + \omega^2)^2 \approx 1/\omega^2 - 2\alpha_m^2 / \omega^4,
\end{aligned} \qquad (11)$$

$$F_7 = \omega/[(\alpha_p^2 + \omega^2)(\alpha_m^2 + \omega^2)] \approx 1/\omega^3,$$

$$F_8 = 1/[\omega(\alpha_p^2 + \omega^2)(\alpha_m^2 + \omega^2)] \approx 0,$$

$$F_9 = 1/[(\alpha_p^2 + \omega^2)(\alpha_m^2 + \omega^2)] \approx 1/\omega^4,$$

$$F_{10} = \omega^2/[(\alpha_p^2 + \omega^2)(\alpha_m^2 + \omega^2)] \approx 1/\omega^2 - (\alpha_p^2 + \alpha_m^2)/\omega^4.$$

Taking into account $\omega = M\Omega$, after substituting (11) into (9), and collecting the corresponding terms, we obtain ([23]):

$$K_{(1)} = M, \tag{12}$$

$$K_{(0)} = \omega_0, \quad \omega_0 = -T(\alpha_2 - \alpha_1), \quad T = (\rho - \tau)/(\beta - \theta), \tag{13}$$

$$K_{(-1)} = -2r_1 r_2 (\cos\varepsilon_{12} + 2T\sin\varepsilon_{12})/M, \tag{14}$$

$$K_{(-3)} = -(1/2)(1 + 4T^2)[\alpha_m^2 r_m^2 - \\ - (1/2)(\alpha_p - \alpha_m)(\alpha_2 - \alpha_1)(r_2^2 - r_1^2)]/M^3, \tag{15}$$

$$K_{(-2)} = \{\omega_0[2r_1 r_2(\cos\varepsilon_{12} + 2T\sin\varepsilon_{12})] - \\ - (1/2)(1 + 4T^2)(\alpha_2 - \alpha_1) r_1 r_2 \sin\varepsilon_{12}\}/M^2, \tag{16}$$

$$K_{(-4)} = (1/2)(1 + 4T^2)[\omega_0(3\alpha_m^2 r_m^2) + \\ + (\alpha_p^2 + \alpha_m^2 + \alpha_p\alpha_m)(\alpha_2 - \alpha_1) r_1 r_2 \sin\varepsilon_{12}]/M^4. \tag{17}$$

These relations are valid for $|\Omega| >> \Omega_{\alpha_p}, \Omega_{\alpha_m}$, where $\Omega_{\alpha_p} = \alpha_p/M$, $\Omega_{\alpha_m} = \alpha_m/M$.

As one can see, expressions (12)–(14) and (5)–(7) for coefficients $K_{(1)}$, $K_{(0)}$, $K_{(-1)}$ of polynomial ω_{beat}^P do not differ: they are identical. Expression (15) for coefficient $K_{(-3)}$ only generalizes the known relation (8) for general case $\alpha_1 \neq \alpha_2$. But expressions (16), (17) for coefficients $K_{(-2)}$, $K_{(-4)}$ are substantially new. According to (16), (17), and relation $\omega_0 = -T(\alpha_2 - \alpha_1)$ in (13), coefficients $K_{(-2)}$ and $K_{(-4)}$ depend on difference $\Delta\alpha = \alpha_2 - \alpha_1$ caused by the factor of inequality of counterpropagating waves amplification due to nonreciprocal resonator losses.

Conclusion

In the paper, as a result of solving the well-known system (1) of laser gyro dynamic equations with accuracy to second order in parameters r_j of counterpropagating waves linear coupling, – the modified exact (in wide range of Ω) analytical expression for counterpropagating waves beat frequency ω_{beat} of uniformly rotating device is proposed. This expression has form (9). The first term in (9) is known from the literature, and describes the null shift of laser gyro frequency characteristic. The second and the third terms in (9) are also known from the literature, and describe two reversible with respect to Ω components of ω_{beat}. But the fourth and the fifth terms in (9) are *new*, and describe two nonreversible components of ω_{beat} which are caused by the factor of inequality $(\alpha_1 \neq \alpha_2)$ of counterpropagating waves amplification due to nonreciprocal resonator losses.

Besides that, in the paper, on the base of use of exact expression (9) for ω_{beat} and auxiliary formulas (11), – the known from the literature relations (12), (13), (14) for coefficients $K_{(1)}$, $K_{(0)}$, $K_{(-1)}$ of the polynomial model (3) of counterpropagating waves beat frequency are confirmed, the modified expression (15) for coefficient $K_{(-3)}$ is presented, and the sought for relations (16), (17) for coefficients $K_{(-2)}$, $K_{(-4)}$ are found. Relations (16), (17) are *new*, and describe the manifestation of the above-mentioned factor of inequality of counterpropagating waves amplification.

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