UDC 629. 7. 051

DOI: https://doi.org/10.20535/0203-3771412021269109

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ALGORITHM OF THE AUTONOMOUS INITIAL ALIGNMENT OF SINS WITH SEQUENTIAL FILTERING

Ua Запропоновано та розглядається алгоритм автономної початкової виставки безплатформної інерціальної навігаційної системи на нерухомій або обмежено рухомій основі. У ньому послідовно використовуються два осереднюючі фільтри Калмана. Перший за сигналами акселерометрів визначає один стовпець матриці направляючих косинусів між осями пов'язаного із об'єктом і географічного супровідного базисів. Другий фільтр по сигналах гіроскопів та отриманих оцінок напрямних косінусів визначає другий стовпець матриці. Третій стовпець визначають із відомих співвідношень. З отриманих напрямних косінусів визначають кути курсу, тангажу і крену. Інформаційні сигнали для вирішення задачі одержують шляхом осереднювання малого інтервалу (5 с) наприкінці перехідного процесу фільтра Калмана. Це дозволяє звести до мінімуму помилки від зміни положення об'єкта протягом часу вимірювань, яке може досягати десятків хвилин. Привабливість алгоритму-незалежність помилок від курсу об'єкта. Показано, що попереднє згладжування сигналів дає підвищення точності. Алгоритм дозволяє визначити кути орієнтації, а по них напрямні косинуси при часі вимірювань, недостатньому для завершення перехідного процесу оцінки фільтром Калмана. Проведені натурні експерименти підтверджують хороші характеристики виставки на нерухомому та вібруючому підставі, досягнуті із застосуванням алгоритму.

En An algorithm for an autonomous initial alignment of a strapdown inertial navigation system on a fixed or limitedly movable base is proposed and considered. It sequentially uses two Kalman averaging filters in. The first one determines one column of the directing cosines matrix (DCM) between the axes of the associated with the object and the geographical accompanying bases by the signals of the accelerometers. The second filter determines the second column of the matrix based on the gyroscope signals and the obtained estimates of the direction cosines. The third column is determined from known ratios. From the resulting DCM, the heading, pitch and roll angles are determined. Information signals for solving the problem are obtained by averaging a small interval (5 s) at the end of the Kalman filter transition process. This allows minimizing errors from changing the position of the object during the measurement time, which can reach tens of minutes. The advantage of the algorithm is the independence of errors from the object heading. It is shown that pre-smoothing of signals improves accuracy. The algorithm makes it possible to determine the orientation angles, and then the direction cosines at a measurement time insufficient to complete the transient evaluation process by the Kalman filter. The conducted field experiments confirm the good characteristics of the alignment on a fixed and vibrating base, achieved using the algorithm.

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Introduction

As you know, strapdown inertial navigation systems (SINS) implement the dead reckoning method, which requires the input of initial conditions in the form of place coordinates and initial speed in the chosen coordinate system.

In the geographic coordinate system, the initial angular position of the object (heading, roll, pitch), latitude can be determined autonomously from the signals of gyroscopes and accelerometers. The initial longitude, altitude, speed are entered from external information carriers. Latitude can also be entered from external meters.

The process of obtaining and entering the named initial conditions of motion is called the initial alignment of the inertial navigation system.

The initial exposure usually places higher demands on the sensing elements than the operating mode of the SINS.

The autonomous initial alignment is sometimes referred to as “coarse”. This name is conditional. It means that after a “coarse” alignment, a more accurate alignment can be held.

The initial alignment mode is characterized by two main parameters – accuracy and time. The accuracy depends on the alignment algorithm (exact or iterative [1 - 4]), systematic and random errors (noise) of gyroscopes and accelerometers. The main problem of coarse alignment is the selection of the useful component of the signal in conditions of very high noise. The noise can be ten times higher than the useful signal [7]. To isolate a useful signal, it may be necessary to record a signal lasting more than an hour. Only after processing this record, the initial alignment algorithm itself is used to determine the direction cosines between the coordinate system associated with the SINS (object) and the geographic coordinate system. From the obtained direction cosines, the initial angles of the heading, roll and pitch are also determined.

To date, a fairly large variety of algorithms for the autonomous initial alignment (we will understand the definition of the initial angular position) has developed. Let us name among them triad algorithms [1, 2], basic algorithm [3 - 6], universal algorithm [7], quaternion algorithms [8, 9], optimization algorithms [7, 8], iterative algorithms [6], exposition algorithms according to zero velocity, constant coordinates, and constant SINS orientation angles [13, 14].

A common disadvantage of all the above algorithms is the need for a thorough preliminary cleaning of the signal from noise.

Problem formulation

In the proposed article, an algorithm is proposed and investigated that combines signal filtering and the initial determination of angular parameters.

The main parameters of the alignment are evaluated - accuracy and time to achieve it.

Alignment algorithm with sequential Kalman filters

Consider the proposed alignment algorithm, which allows you to determine the angular coordinates from noisy signals.

The matrix of direction cosines C^{gb} between the axes (Fig. 1) of the geographic accompanying basis $\xi\eta\zeta$ (let's denote it g) and the xyz basis associated with the SINS (let's denote it b) corresponds to the heading ψ, pitch ϑ and roll γ angles and has the form [5]

***Table* 1.**

Direction cosines

C^{gb}	x	y	z
ξ	$c_{11} = \cos\gamma_0 \cos\psi_0 + \sin\gamma_0 \sin\psi_0 \sin\vartheta_0$;	$c_{12} = \cos\vartheta_0 \sin\psi_0$	$c_{13} = \cos\psi_0 \sin\gamma_0 - \sin\psi_0 \cos\gamma_0 \sin\vartheta_0$;
η	$c_{21} = -\cos\gamma_0 \sin\psi_0 + \sin\gamma_0 \cos\psi_0 \sin\vartheta_0$;	$c_{22} = \cos\vartheta_0 \cos\psi_0$;	$c_{23} = -\sin\psi_0 \sin\gamma_0 - \cos\gamma_0 \cos\psi_0 \sin\vartheta_0$;
ς	$c_{31} = -\cos\vartheta_0 \sin\gamma_0$;	$c_{32} = \sin\vartheta_0$;	$c_{33} = \cos\vartheta_0 \cos\gamma_0$.

The task of the initial alignment is to determine these angles and direction cosines.

The direction cosines $c_{31}\, c_{32}\, c_{33}$ can be determined from the vector-matrix equation

$$\begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} = \begin{bmatrix} g & 0 & 0 \\ 0 & g & 0 \\ 0 & 0 & g \end{bmatrix} \cdot \begin{bmatrix} c_{31} \\ c_{32} \\ c_{33} \end{bmatrix}. \qquad (1)$$

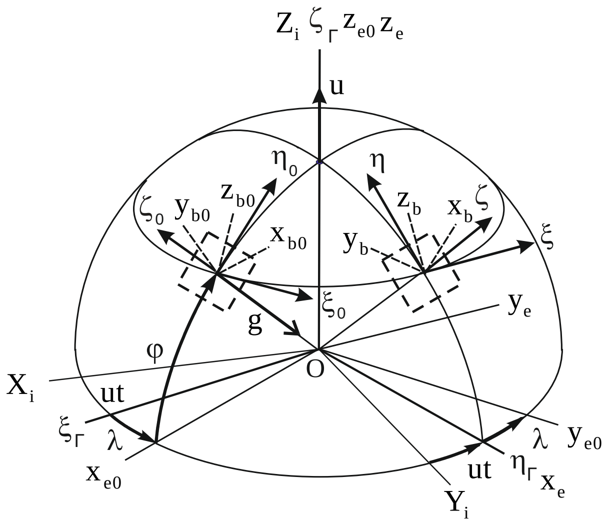


Fig. 1. Bases and their displacement

Here a_x, a_y, a_z are apparent accelerations measured by accelerometers, g is the acceleration due to gravity. Let's write the measurement vector

$$\vec{y}_a = [a_x \quad a_y \quad a_z]^T .$$

$\vec{x}_a = [c_{31}\ c_{32}\ c_{33}]^T$ - the vector of the desired parameters (the vector of the state variables).), $H_a = diag(g\ g\ g)$ – matrix of measurements.

Apply the Kalman averaging filter algorithm [7, 17].

We write the equation of dynamics (state) for accelerometers in the form

$$\vec{x}_{ak+1} = \vec{x}_{ak} + \vec{w}_{ak} .$$

We assume, that the random components of the vector $\vec{w}_a$ disturbances $\sigma_{ai} \cdot w_{1i},\ \ i = x, y, z$ are "white noise", σ_{ai} – the standard deviation (std) of the accelerometer signals, $\vec{w}_1$ - generating "white noise" of unit intensity.

We write the measurement equation in the form

$$\vec{y}_{ak} = \mathbf{H}_a \vec{x}_{ak} + \vec{v}_{ak},$$

where $\vec{v}_a$ – measurement noise.

We apply the discrete linear Kalman filter algorithm [5, 7]

$$\breve{\mathbf{x}}_{a\ k+1} = \mathbf{F_k}\,\hat{\mathbf{x}}_{a\ k}$$, (further index "*a*" is omitted)

$$\mathbf{K_k} = \breve{\mathbf{P}}_\mathbf{k}\mathbf{H}_\mathbf{k}^\mathbf{T}\left[\mathbf{H_k}\breve{\mathbf{P}}_\mathbf{k}\mathbf{H}_\mathbf{k}^\mathbf{T} + \mathbf{R}_k\right]^{-1},$$

$$\hat{\mathbf{x}}_\mathbf{k} = \breve{\mathbf{x}}_\mathbf{k} + \mathbf{K_k}\left[\mathbf{y_k} - \mathbf{H_k}\breve{\mathbf{x}}_\mathbf{k}\right],$$

$$\hat{\mathbf{P}}_\mathbf{k} = \left[\mathbf{I} - \mathbf{K_k}\mathbf{H_k}\right]\breve{\mathbf{P}}_k ,$$

$$\breve{\mathbf{P}}_\mathbf{k+1} = \mathbf{F_k}\hat{\mathbf{P}}_\mathbf{k}\mathbf{F}_\mathbf{k}^\mathbf{T} + \mathbf{Q}_k .$$

From the equation of dynamics we see that the transition matrix F is the identity matrix I.

Correlation matrix of measurements

$R_a = kR_a * diag(\sigma_{ax}^2\ \sigma_{ay}^2\ \sigma_{az}^2)$, kR_a - adjustment factor;

correlation matrix of perturbations

$Q_a = kQ_a * diag(\sigma_{ax}^2\ \sigma_{ay}^2\ \sigma_{az}^2)$, kQ_a - adjustment factor.

At the output of the filter we obtain estimates of the direction cosines $\hat{c}_{31}\ \hat{c}_{32}\ \hat{c}_{33}$.

Let us proceed to the preparation of the model of the second Kalman filter to evaluate $c_{21}\ c_{22}\ c_{23}$.

We use the following expressions

$$\begin{aligned} \omega_x - \omega_\zeta \hat{c}_{31} &= \omega_\xi c_{11} + \omega_\eta c_{21}, \\ \omega_y - \omega_\zeta \hat{c}_{32} &= \omega_\xi c_{12} + \omega_\eta c_{22}, \\ \omega_z - \omega_\zeta \hat{c}_{33} &= \omega_\xi c_{13} + \omega_\eta c_{23}, \end{aligned} \tag{2}$$

where ω_x, ω_y, ω_z - absolute angular velocities measured by gyroscopes; ω_ξ, ω_η, ω_ζ - components of the angular velocity of the geographical accompanying basis [5].

Let us substitute into (2) the formulas for the ratios of the direction cosines

$$c_{11} = c_{22}c_{33} - c_{23}c_{32}, \quad c_{12} = c_{23}c_{31} - c_{21}c_{33}, \quad c_{13} = c_{21}c_{32} - c_{22}c_{31}. \tag{3}$$

Get

$$\begin{aligned} \omega_x - \omega_\zeta \hat{c}_{31} &= \omega_\xi (c_{22}c_{33} - c_{23}c_{32}) + \omega_\eta c_{21}, \\ \omega_y - \omega_\zeta \hat{c}_{32} &= \omega_\xi (c_{23}c_{31} - c_{21}c_{33}) + \omega_\eta c_{22}, \\ \omega_z - \omega_\zeta \hat{c}_{33} &= \omega_\xi (c_{21}c_{32} - c_{22}c_{31}) + \omega_\eta c_{23}. \end{aligned} \tag{4}$$

We write system (4) in the vector-matrix form

$$\vec{y}_\omega = \mathbf{H} \cdot \vec{x}_\omega. \tag{5}$$

Here $\vec{x}_\omega = [c_{21}\ c_{22}\ c_{23}]^T$ - the vector of the desired parameters, $\vec{y}_\omega = [\omega_x - \omega_\zeta \hat{c}_{31}, \omega_y - \omega_\zeta \hat{c}_{32}, \omega_z - \omega_\zeta \hat{c}_{33}]^T$ - the measurement vector.

Measurement matrix $$\mathbf{H} = \begin{bmatrix} \omega_\eta & \omega_\xi \hat{c}_{33} & -\omega_\xi \hat{c}_{32} \\ -\omega_\xi \hat{c}_{33} & \omega_\eta & \omega_\xi \hat{c}_{31} \\ \omega_\xi \hat{c}_{32} & -\omega_\xi \hat{c}_{31} & \omega_\eta \end{bmatrix} / \tag{6}$$

We write the equation of dynamics (state) for gyros and equation of measurement in the form

$$\vec{x}_{\omega\ k+1} = \vec{x}_{\omega\ k} + \vec{w}_{\omega\ k},$$
$$\vec{y}_\omega = \mathbf{H} \cdot \vec{x}_\omega + \vec{v}_\omega,$$

where $\vec{w}_\omega$ and $\vec{v}_\omega$ - disturbance and measurement noise.

Correlation matrix of measurements

$$R_\omega = kR_\omega \cdot diag(\sigma^2_{\omega x}\ \sigma^2_{\omega y}\ \sigma^2_{\omega z}), \quad kR_\omega \text{ – adjustment factor;}$$

correlation matrix of perturbations

$$Q_\omega = kQ_\omega \cdot diag(\sigma^2_{\omega x}\ \sigma^2_{\omega y}\ \sigma^2_{\omega z}), \quad kQ_\omega \text{ adjustment factor.}$$

Initial values of error correlation matrices $P_a = 0_{3\times3}$. $P_\omega = 1e - 6 \cdot I_{3\times3}$.

Applying the Kalman filter algorithm, we obtain the estimates $\hat{c}_{21}\ \hat{c}_{22}\ \hat{c}_{23}$.

Then, according to formulas (4), we obtain heading ψ, pitch ϑ and roll γ angles. The block diagram of the algorithm can be presented in Fig. 2 (Fig. 2, are marked $x1 \equiv x_a$, $x2 \equiv x_\omega$, $y1 \equiv y_a$, $y2 \equiv y_\omega$).

As can be seen from the algorithm, the direction cosines are determined using Kalman filters, and then the orientation angles are determined from them. It can be seen from the diagram that the algorithm can also work with rectilinear uniform motion.

Simulation

Well-known mathematical models of gyroscope and accelerometer signals are adopted for modeling.

Signal model of the gyroscopes triad

Let's imagine a model of a of gyroscopes triad:

$$\vec{u}_\omega = \cdot R_\omega(\omega) \cdot D_\omega \cdot \vec{\omega} + B \cdot \vec{a} + \vec{u}_{\omega 0} + \vec{n}_{u\omega}.$$

$\vec{u}_\omega = \begin{bmatrix} u_{\omega x} & u_{\omega y} & u_{\omega z} \end{bmatrix}^T$ - output signal of a gyroscopes triad in an oblique coordinate system (SC) associated with the measuring axes in the dimension of angular velocity;

$\vec{\omega} = \begin{bmatrix} \omega_x & \omega_y & \omega_z \end{bmatrix}^T$ - effective angular velocity in the orthogonal SC associated with the landing surfaces of the measuring module (MM);

$\vec{u}_{\omega 0} = \begin{bmatrix} u_{\omega 0x} & u_{\omega 0y} & u_{\omega 0z} \end{bmatrix}^T$ - offsets of the gyroscopes zero signal;

$R_\omega = \begin{bmatrix} R_{\omega x}(\omega) & 0 & 0 \\ 0 & R_{\omega y}(\omega) & 0 \\ 0 & 0 & R_{\omega z}(\omega) \end{bmatrix}$ – matrix of scale factors (MC) of the triad. The output signal may be non-linear with respect to the actual angular velocity.

D_ω - orientation (non-orthogonality) matrix that binds the SC formed by the measuring axes of the gyroscope unit to the SC associated with the landing surfaces of the MI;

B – matrix of gyroscopes to acting accelerations.

Signal model of accelerometers triad

The accelerometer signal can be represented by model

$\vec{u}_a = \cdot K(a) \cdot D_a \cdot \vec{a} + \vec{u}_{a0} + \vec{n}_{ua}$,

$\vec{u}_a = \begin{bmatrix} u_{ax} & u_{ay} & u_{az} \end{bmatrix}^T$ - the output signal of the gyroscopes triad,

$\vec{a} = \begin{bmatrix} a_X & a_Y & a_Z \end{bmatrix}^T$ - the effective apparent acceleration,

$\vec{u}_{a0} = \begin{bmatrix} u_{a0x} & u_{a0y} & u_{a0z} \end{bmatrix}^T$ - offsets of the zero signal of accelerometers;

$K_a(a) = \begin{bmatrix} K_{ax}(a) & 0 & 0 \\ 0 & K_{ay}(a) & 0 \\ 0 & 0 & K_{az}(a) \end{bmatrix}$ – matrix of scale factors (MC) of the triad.

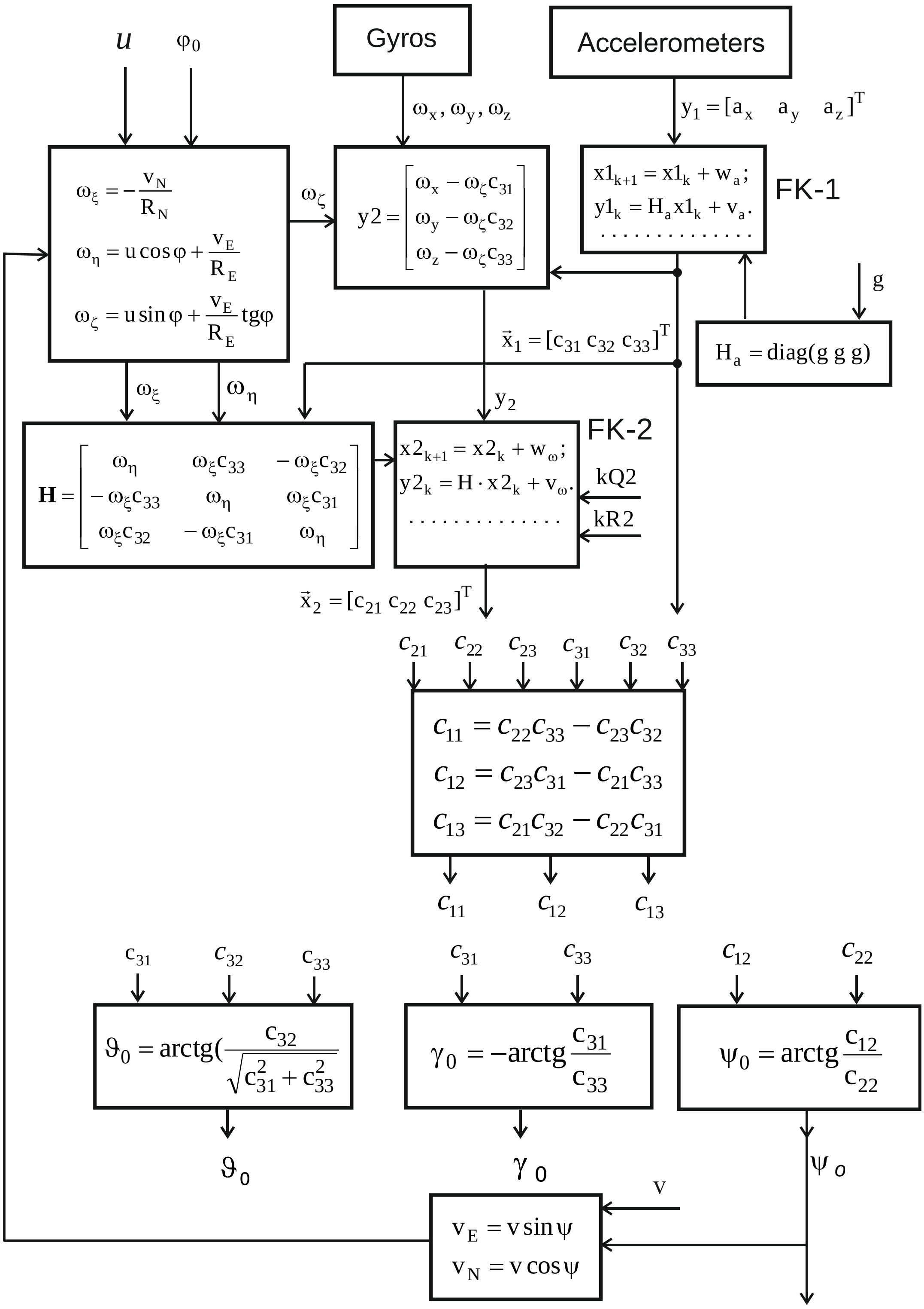


Fig. 2. Initial alignment algorithm with Kalman filters (FK2)

The output signal may be non-linear with respect to the actual acceleration.

D_a - orientation matrix of the measuring axes (non-orthogonality),

B – matrix of sensitivity of gyroscopes to acting accelerations.

$\vec{n}_{ua} = [n_{ax}\ n_{ay}\ n_{az}]^T$ - vector of random components of accelerometer signals.

Simplified models

Matrices of transmission coefficients (scale factors) and distortions of the sensitivity axes can be obtained as a result of calibration and multiplied before performing operational measurements, which makes it possible to simplify the models:

gyroscopes

$$\begin{bmatrix} u_{\omega x} \\ u_{\omega y} \\ u_{\omega z} \end{bmatrix} = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix} \cdot \begin{bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} + \begin{bmatrix} u_{\omega x0} \\ u_{\omega y0} \\ u_{\omega z0} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} n_{\omega x} \\ n_{\omega y} \\ n_{\omega z} \end{bmatrix}, \qquad (7)$$

accelerometers

$$\begin{bmatrix} u_{ax} \\ u_{ay} \\ u_{az} \end{bmatrix} = \begin{bmatrix} k_{11} & k_{12} & k_{13} \\ k_{21} & k_{22} & k_{23} \\ k_{31} & k_{32} & k_{33} \end{bmatrix} \cdot \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix} + \begin{bmatrix} u_{ax0} \\ u_{ay0} \\ u_{az0} \end{bmatrix} + \begin{bmatrix} n_{ax} \\ n_{ay} \\ n_{az} \end{bmatrix}. \qquad (8)$$

The models do not take into account temperature errors assuming operation in a steady temperature regime.

Simulation results

When modeling the alignment algorithm FK2, the results shown in fig. 3, fig. 4. On fig. 3 are: **fi0** – latitude, **tet** – pitch angle, **gam** – roll angle, **hpsi -** heading change step, **slom** – standart deviation (std) of gyroscope noise, **dfi** – latitude error, **om0** – gyroscope zero offset, **ac0** – accelerometer zero offset , **h** - sample step, **kA** - yaw amplitude, **slac** - standart deviation of the accelerometer noise, **nort** - maximum value of the non-orthogonality angle of the block axes, dsom - instability of the gyroscope scale factor, dsac - instability of the accelerometer scale factor, $Q1 \equiv kQ_a, R1 \equiv kR_a$ - Kalman filter tuning coefficients of horizontal channels, $Q2 \equiv kQ_\omega, R2 \equiv kR_\omega$ – Kalman filter tuning coefficients of the heading channel, tizm – signal measurement time, tsr – signal averaging time, **rms psi** – root mean square heading error.

Since gyroscopes measure the absolute angular velocity relative to the inertial frame , and during the initial alignment, their position in the inertial coordinate system changes (fig. 1), the measured projections of the angular velocity change during the measurement. An estimate of the error from such a change is given in [6].

To reduce this error, it is necessary to reduce the signal processing (filtering) interval. In the proposed algorithm, in addition to the Kalman filtering, simple signal averaging is carried out over the last 5 s of the Kalman filter transient.

If you pre-clear the input signals, for example, by moving average, you can get the result shown in fig. 4.

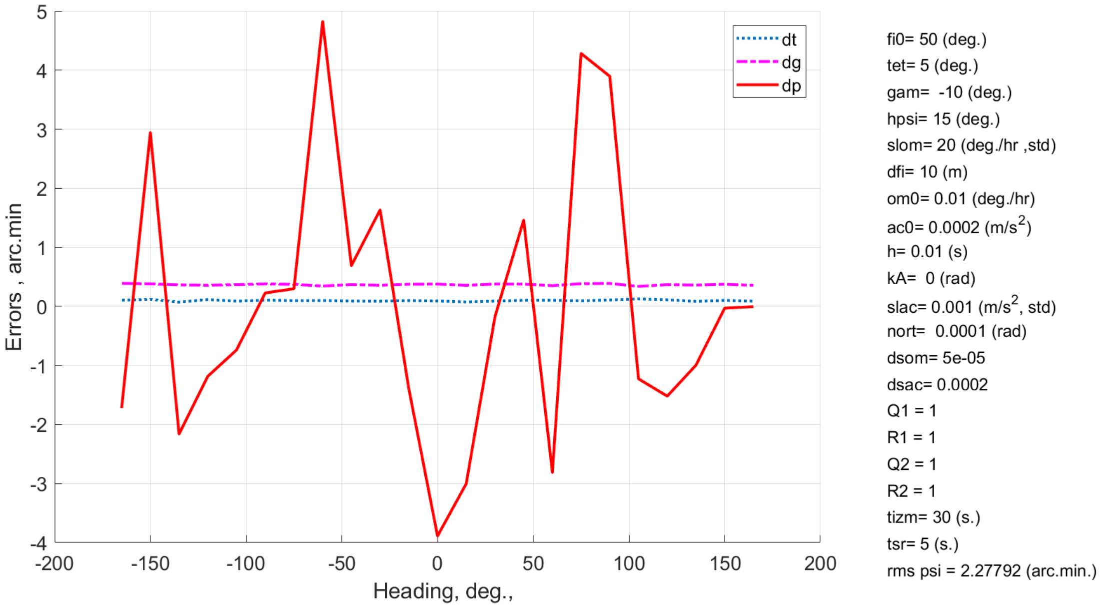
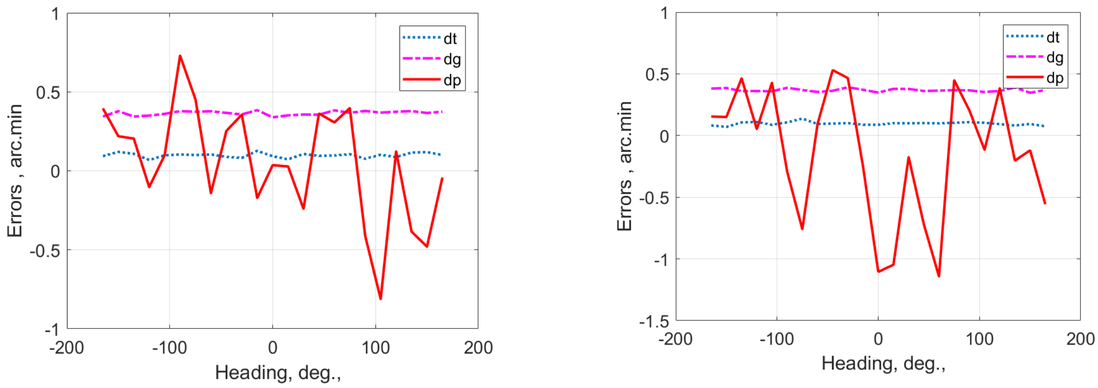


Fig. 3. Errors of the initial alignment



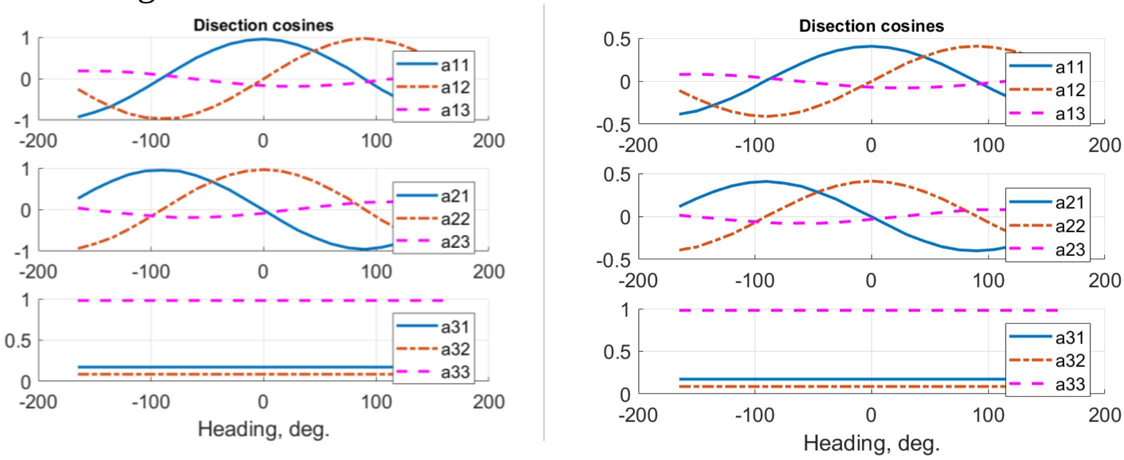
a) *b*)

Fig. 4. Errors in the alignment with algorithm with preliminary noise reduction of gyroscopes up to 2 deg/hr (std): *a*) time of measur. 30 s, rmpsi=0,36 arcmin; *b*) measure time 10 s, rmspsi=0,54 arc min

From fig. 4 it can be seen that even with a decrease in the measurement time to 10 s, heading alignment errors can have an acceptable value. As can be seen from fig. 5, the 10 second direction cosine estimation transient does not reach a steady state estimate. However, since this is the case for all direction cosines, and the angles are calculated in terms of the arc tangents of the direction cosine ratios, the end result may be acceptable. The direction cosines can then be found using formulas (1).

Increasing the measuring interval improves the accuracy. If when measuring in tizm = 30 s we got rms psi = 2,5 arc min., then when measuring in tizm = 300 s rms psi= 0,67 arc min.

The advantage of the algorithm is also that the magnitude of the error is practically independent of the heading, in contrast to the basic, universal, optimization algorithms.



a) *b*)

Fig. 5. Estimates of direction cosines: *a*) measurement time 30 s, *b)* measurement time 10 s

Comparison fig. 3, fig. 4 (rmspsi) shows that preliminary smoothing of noise can give a noticeable positive result.

Field experiments

Table 2 shows the results of 11 field experiments on three different headings using the initial alignment algorithm with Kalman filters (FK2).

In the experiments, a measuring unit with the following characteristics was used:

- zero instability of gyroscopes – 0,01 deg./hr;
- instability of the scale factor of gyroscopes - 50 ppm (5e-5);
- gyroscope signal noise – 9…35 deg./hr [7, 17] (sum of quantization noise, white noise etc);
- zero instability of accelerometers – 2e-3 m/s^2;
- instability of the scale factor of accelerometers – 200 ppm.
- accelerometer signal noise – (4…5) 10^{-3} m/s^2 [7, 17];

The experiment was carried out on a stationary car with the engine off. Such a base can be assumed to be conditionally immovable, since wind disturbances, ground vibrations, and movements in the car could take place. The measurements were carried out for tizm = 30 s with a decrease in the measurement matrix R by a factor of 100 compared to the initially specified one. Decreasing R contributes to the acceleration of the transient process. The final result in each experiment is obtained as an average of the last 5 s (tsr) of the filter transient (fig. 6).

The root-mean-square error (rms) in 11 experiments was 2,5 arcsec. min. Increasing the estimation time, as usual, improves accuracy.

With the improvement of Kalman filters [15, 16], one can also expect an additional positive result.

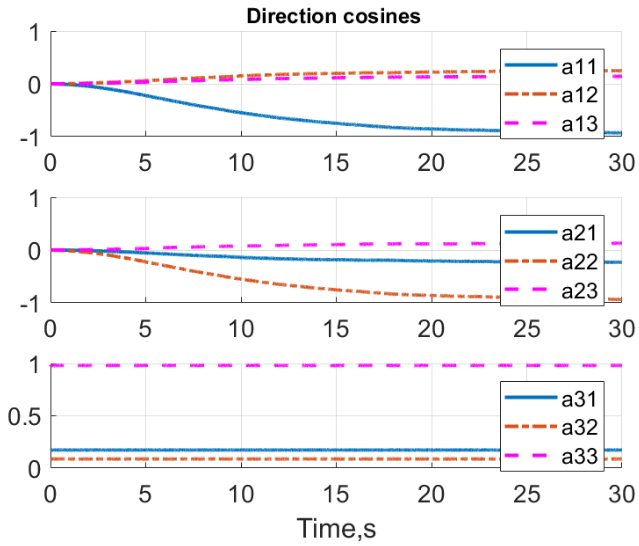


Fig. 6. Transition evaluation process direction cosines

***Table* 2.**

Results of experiments with the engine turned off

Heading		Algorithm – FK2
Measurement time – 30 s, Average of the last tosr=5 s Tuning – kQ2=1, kR2=0,01		
Ideal., degrees	Instrum., degrees	Error, arc.min.
356,466	356,503	2,252
356,466	356,508	2,528
356,466	356,449	-1,006
356,466	356,435	-1,864
356,466	356,422	-2,614
356,466	356,466	-0,004
358,415	358,409	-0,394
358,415	358,304	-6,694
90,179	90,205	1,531
90,179	90,189	0,599
90,179	90,195	0,974
	Rms=	2,548

Tuning the Kalman filter to determine the heading angle by changing the perturbation matrices *Q* and measurements matrices *R* during simulation may not give a guaranteed positive result, since in each experiment the optimal tuning coefficients *kQ* and kR turn out to be different. Tuning coefficients also lead to a slight bias in estimates.

The results of the alignment are shown in Table 3, where the left side was obtained with the vehicle stopped and the engine turned off, and the right side was obtained with the engine turned on.

***Table* 3.**

Results of the alignment with preliminary averaging of signals

Engine turned off				Engine turned on			
Heding	Algorithm – FK2			Heding	Algorithm - FK2		
	Measurement time – 60 s Average of the last tosr=15 s Tuning – kQ=1, kR=0,01				Measurement time – 60 s Average of the last tosr=15 s Tuning – kQ=1, kR=0,01		
№	Ideal., degrees	Instrum., degrees	Error, arc.min.	№	Ideal., degrees	Instrum., degrees	Error, arc.min
1	356,466	356,534	4,106	1	356,466	356,424	-2,518
2	356,466	356,486	1,244	2	356,466	356,472	0,386
3	356,466	356,499	2,000	3	356,466	356,418	-2,842
4	356,466	356,464	-0,082	4	356,466	356,433	-1,966
5	356,466	356,469	0,200	5	356,466	356,330	-8,146
6	356,466	356,490	1,484	6	356,466	356,446	-1,174
7	358,415	358,388	-1,612	7	358,415	358,416	0,032
8	358,415	358,469	3,248	8	358,415	358,388	-1,606
		Rms8 =	1,854			Rms8 =	3,331

As previously mentioned, a noticeable increase in accuracy can be obtained by pre-smoothing the signals. A result close to it will be obtained with an increase in the measurement interval or an increase in the averaging interval at the end of the steady-state transient process (tsr), on which the output result is determined.

Conclusions

The presented algorithm solves the problem of an autonomous initial alignment on a fixed or conditionally fixed base. It provides sufficient smoothing of the signal noise, high accuracy with a fairly short measurement time.

To calculate the initial angular position parameters, it is better to use a steady-state estimation process. However, even with a short measurement time, when the transient evaluation process has not yet been completed, acceptable results can be obtained.

Heading initial alignment errors do not depend on the object's heading.

To increase the accuracy, you can increase the measurement time.

To reduce errors from changes in the angular velocities of an object measured by gyroscopes, it is recommended to obtain the final result by averaging the transient process of the Kalman filter over a small interval at the end of the process.

The above algorithms make it possible to almost completely eliminate the effect of measurement noise. The final accuracy of the alignment depends on the systematic errors of the meters caused by various reasons.

In the future, it is useful to consider the effect of changing the frequency of measurements, a wider variation in time intervals. An additional positive effect can be obtained by using refinement filters (for example, with the reverse calculation). It is useful to study the features and parameters of preliminary smoothing of signals. It is of interest to consider the characteristics of the algorithm for autonomous determination of latitude, as well as for a constant speed.

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