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GEOMETRICAL SCALE MULTIPLIER OF THE RING LASER GYRO WITH RESONATOR CONTAINING A DIELECTRIC MEDIUM

Ua Як показує аналіз літератури, існує щонайменше шість якісно різних аналітичних виразів для геометричного масштабного множника M_g лазерного гіроскопа з плоским N-дзеркальним резонатором, що містить уздовж всього периметра L тверде діелектричне середовище з показником заломлення $n = (\varepsilon_r \mu_r)^{1/2}$. Відповідно до одного з таких виразів, параметр M_g обернено пропорційний величині n. У статті, – цей вираз для M_g підтверджено.

En As analysis of the literature shows, there are at least six qualitatively different analytical expressions for geometrical scale multiplier M_g of the ring laser gyro with a planar N-mirror resonator containing along all perimeter L a rigid dielectric medium with index of refraction $n = (\varepsilon_r \mu_r)^{1/2}$. According to one of these expressions, the parameter M_g is inversely proportional to quantity n. In the paper, – such expression for M_g is confirmed.

Introduction

One of the important metrological parameters of the ring laser gyro ([1] - [6]) with a planar N-mirror resonator ($N \geq 3$) is the so-called geometrical scale multiplier M_g. It is the coefficient of proportionality between the angular velocity Ω_z, with which the gyro rotates in the inertial space about its sensitivity axis $\hat{z}$, and the difference $\Delta\omega = \omega_2 - \omega_1$ between frequencies of counterpropagating in its resonator waves:

$$\Delta\omega = M_g \Omega_z \quad (\Omega_z = \vec{\Omega} \cdot \hat{z}). \tag{1}$$

In these formulas, Ω_z is the projection of vector of the laser gyro absolute angular velocity $\vec{\Omega}$ onto the unit vector $\hat{z}$ which is orthogonal to resonator plane. The parameter M_g in (1) is called “geometrical” because it depends mainly on geometry of the laser gyro resonator.

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If the laser gyro has *an empty* N-mirror resonator of arbitrary shape (with perimeter L and area A) which provides generation of radiation linearly polarized in the sagittal plane, then the expression for parameter M_g has well-known and adopted by all researches form

$$M_g = M_{g0} = \frac{8\pi A}{\lambda_0 L}, \tag{2}$$

where λ_0 is the wavelength in vacuum related to the central of active medium emission line frequency ω_0 by the formula $\lambda_0 = 2\pi c / \omega_0$.

But another situation is observed in general case, when the laser gyro resonator *is not empty* and contains a rigid dielectric medium with length d and index of refraction $n = (\varepsilon_r \mu_r)^{1/2}$, where ε_r and μ_r are the relative permittivity and permeability of the medium, respectively.

In order to compare all results in this field obtained in numerous works by various authors, it will be convenient to consider (as in [17]) the special mutual case, $d = L$, when the dielectric medium is located inside the laser gyro resonator along all perimeter L, and the active medium is concentrated only within a infinitely thin layer.

Then expressions for parameter M_g under such condition will have the following forms:

– according to [2] (case $\mu_r = 1$), [7]–[19], and [20] (see Section IV therein),

$$M_g = M_{g0}\left(\frac{1}{n}\right); \tag{3}$$

– in accordance with [20] (see Section V therein), and [21]–[22],

$$M_g = M_{g0}\left(\frac{\mu_r}{\varepsilon_r}\right)^{1/2}; \tag{4}$$

– according to [23],

$$M_g = M_{g0}\frac{(1+n^2)}{2n}; \tag{5}$$

– in accordance with [4] (case $\mu_r = 1$) and [24],

$$M_g = M_{g0}\left(\frac{1}{n^2}\right); \tag{6}$$

– according to [25] (case $\mu_r = 1$),

$$M_g = M_{g0}\, n; \tag{7}$$

– and, finally, as it is mentioned in [25] about results of [26],

$$M_g = M_{g0}. \tag{8}$$

It must be noted that expressions (3) – (8) for parameter M_g are approximate. They are obtained in a linear with respect to Ω ($\Omega = \left|\vec{\Omega}\right|$) approximation.

Remark: Comprehensive list of papers on this theme is presented in [17].

As one can see from formulas (3) – (8), there are at least six qualitatively different analytical expressions for geometrical scale multiplier M_g of the ring laser gyro with a planar N-mirror resonator containing along all perimeter L a rigid dielectric medium with index of refraction $n = (\varepsilon_r \mu_r)^{1/2}$.

The goal of this work is to confirm the known expressions (3) for parameter M_g. In order to do it with a greater level of authenticity, a new approach to derivating the system of Maxwell's equations in a uniformly rotating dielectric medium will be used. All calculations will be performed with accuracy approximated to first order in Ω.

Derivating the system of Maxwell's equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ in a uniformly rotating dielectric medium

Consider the inertial frame of reference $\{\hat{x}'\ \hat{y}'\ \hat{z}'\}$ with origin in point O', spatial rectangular coordinates x', y', z', and time coordinate t'. Radius-vector $\vec{r}'$ of a given observation point S' in such frame is $\vec{r}' = x'\hat{x}' + y'\hat{y}' + z'\hat{z}'$, where $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ are the unit vectors. In such inertial frame, in the absence of free currents and charges, the system of Maxwell's equations in a dielectric medium has well-known form:

$$\begin{aligned}
&\vec{\nabla}'\times\vec{E}' + \frac{\partial \vec{B}'}{\partial t'} = 0, \\
&\vec{\nabla}'\cdot\vec{B}' = 0, \\
&\vec{\nabla}'\times\vec{B}' - \frac{1}{c^2}\frac{\partial \vec{E}'}{\partial t'} = \mu_0\left(\frac{\partial \vec{P}'}{\partial t'} + \vec{\nabla}'\times\vec{M}'\right), \\
&\vec{\nabla}'\cdot\vec{E}' = -\frac{1}{\varepsilon_0}\vec{\nabla}'\cdot\vec{P}'.
\end{aligned} \tag{9}$$

System (9) is written here in the SI units. In this system, $\vec{E}'$ and $\vec{B}'$ are the electromagnetic field vectors; $\vec{P}'$ and $\vec{M}'$ are the polarization and magnetization vectors of a dielectric medium; $\vec{\nabla}' = \hat{x}'(\partial/\partial x') + \hat{y}'(\partial/\partial y') + \hat{z}'(\partial/\partial z')$ is the operator of spatial differentiation; $\partial/\partial t'$ is the operator of time differentiation; $c = (\varepsilon_0 \mu_0)^{-1/2}$ is the speed of light in vacuum (ε_0 and μ_0 are its permittivity and permeability, respectively).

In system (9), vectors $\vec{E}'$, $\vec{B}'$, $\vec{P}'$, $\vec{M}'$ are related via expressions

$$\vec{P}' = (\varepsilon_r - 1)\varepsilon_0 \vec{E}', \qquad \vec{M}' = \left(1 - \frac{1}{\mu_r}\right)\frac{\vec{B}'}{\mu_0}. \tag{10}$$

Substituting (10) into (9) yields the standard system of Maxwell's equations for electromagnetic field vectors $\vec{E}'$ and $\vec{B}'$ in a nonrotating dielectric medium:

$$\begin{aligned}
&\vec{\nabla}' \times \vec{E}' + \frac{\partial \vec{B}'}{\partial t'} = 0, \\
&\vec{\nabla}' \cdot \vec{B}' = 0, \\
&\vec{\nabla}' \times \frac{\vec{B}'}{\mu_r} - \frac{1}{c^2}\frac{\partial \varepsilon_r \vec{E}'}{\partial t'} = 0, \\
&\vec{\nabla}' \cdot \varepsilon_r \vec{E}' = 0.
\end{aligned} \tag{11}$$

So the main question of this section is: what form will system (11) have in a uniformly rotating dielectric medium?

To answer the question, consider a uniformly rotating frame of reference $\{\hat{x}\ \hat{y}\ \hat{z}\}$ with origin in point O ($O = O'$), spatial rectangular coordinates x, y, z, and time coordinate t ($t = t'$). Radius-vector $\vec{r}$ of a given observation point S in such frame is $\vec{r} = x\hat{x} + y\hat{y} + z\hat{z}$, where $\hat{x}$, $\hat{y}$, $\hat{z}$ are the unit vectors. At initial moment $t = t' = 0$, the unit vectors $\hat{x}$, $\hat{y}$, $\hat{z}$ of such rotating frame coincide with the unit vectors $\hat{x}'$, $\hat{y}'$, $\hat{z}'$ of the inertial one, radius-vector $\vec{r}$ of observation point S coincides with radius-vector $\vec{r}'$ of point S', and $S = S'$.

Let us introduce in such rotating frame the electromagnetic field vectors $\vec{E}$ and $\vec{B}$, the polarization and magnetization vectors $\vec{P}$ and $\vec{M}$, the operator of spatial differentiation $\vec{\nabla} = \hat{x}(\partial/\partial x) + \hat{y}(\partial/\partial y) + \hat{z}(\partial/\partial z)$, and the operator of time differentiation $\partial/\partial t$. We will suppose that vectors $\vec{E}$, $\vec{B}$, $\vec{P}$, $\vec{M}$ in a rotating frame are related (as in the inertial one) via expressions

$$\vec{P} = (\varepsilon_r - 1)\,\varepsilon_0\,\vec{E}, \qquad \vec{M} = \left(1 - \frac{1}{\mu_r}\right)\frac{\vec{B}}{\mu_0}. \tag{12}$$

Let the frame of reference $\{\hat{x}\ \hat{y}\ \hat{z}\}$, at initial moment $t = t' = 0$, begins to rotate with respect to the inertial one $\{\hat{x}'\ \hat{y}'\ \hat{z}'\}$ with angular velocity $\vec{\Omega} = \Omega_x\,\hat{x} + \Omega_y\,\hat{y} + \Omega_z\,\hat{z}$. So the formulas of transformation of coordinates between these two frames are

$$dt = dt', \qquad d\vec{r} = d\vec{r}\,' - \vec{v}\,dt', \\ \vec{v} = \vec{\Omega}\times\vec{r} = v_x\,\hat{x} + v_y\,\hat{y} + v_z\,\hat{z}, \tag{13}$$

or, in scalar form,

$$dt = dt', \quad dx = dx' - v_x\,dt', \quad dy = dy' - v_y\,dt', \quad dz = dz' - v_z\,dt', \tag{14}$$

where

$$v_x = \Omega_y\,z - \Omega_z\,y, \quad v_y = \Omega_z\,x - \Omega_x\,z, \quad v_z = \Omega_x\,y - \Omega_y\,x. \tag{15}$$

Let us find relation between operators $\vec{\nabla}'$ and $\vec{\nabla}$. Taking into account (14) and (15), with the help of formulas

$$\frac{\partial}{\partial x'} = \frac{\partial t}{\partial x'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial x'}\frac{\partial}{\partial x} + \frac{\partial y}{\partial x'}\frac{\partial}{\partial y} + \frac{\partial z}{\partial x'}\frac{\partial}{\partial z}, \\ \frac{\partial}{\partial y'} = \frac{\partial t}{\partial y'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial y'}\frac{\partial}{\partial x} + \frac{\partial y}{\partial y'}\frac{\partial}{\partial y} + \frac{\partial z}{\partial y'}\frac{\partial}{\partial z}, \\ \frac{\partial}{\partial z'} = \frac{\partial t}{\partial z'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial z'}\frac{\partial}{\partial x} + \frac{\partial y}{\partial z'}\frac{\partial}{\partial y} + \frac{\partial z}{\partial z'}\frac{\partial}{\partial z}, \tag{16}$$

we obtain

$$\frac{\partial}{\partial x'} = \frac{\partial}{\partial x}, \qquad \frac{\partial}{\partial y'} = \frac{\partial}{\partial y}, \qquad \frac{\partial}{\partial z'} = \frac{\partial}{\partial z}, \tag{17}$$

so

$$\vec{\nabla}' = \vec{\nabla}. \tag{18}$$

Let us find relation between operators $\partial/\partial t'$ and $\partial/\partial t$. Taking into account (14) and (15), with the help of formula

$$\frac{\partial}{\partial t'} = \frac{\partial t}{\partial t'}\frac{\partial}{\partial t} + \frac{\partial x}{\partial t'}\frac{\partial}{\partial x} + \frac{\partial y}{\partial t'}\frac{\partial}{\partial y} + \frac{\partial z}{\partial t'}\frac{\partial}{\partial z}, \tag{19}$$

we get

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} - (\vec{v}\cdot\vec{\nabla}). \qquad (20)$$

The frame of reference $\{\hat{x}\ \hat{y}\ \hat{z}\}$ rotates with respect to the inertial one $\{\hat{x}'\ \hat{y}'\ \hat{z}'\}$ with angular velocity $\vec{\Omega}$. So expression (20) must be supplemented with the term $\vec{\Omega}\times$. As a result,

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} - (\vec{v}\cdot\vec{\nabla}) + \vec{\Omega}\times. \qquad (21)$$

According to (21), for any given vector $\vec{G}$ $(\vec{G} = \vec{E}, \vec{B})$,

$$\frac{\partial\vec{G}}{\partial t'} = \frac{\partial\vec{G}}{\partial t} - (\vec{v}\cdot\vec{\nabla})\vec{G} + \vec{\Omega}\times\vec{G}. \qquad (22)$$

For subsequent calculations, it will be convenient to rewrite (22) in other, but equivalent form. Let us consider the following identity:

$$\vec{\nabla}\times(\vec{v}\times\vec{G}) = (\vec{G}\cdot\vec{\nabla})\vec{v} - (v\cdot\vec{\nabla})\vec{G} + \vec{v}(\vec{\nabla}\cdot\vec{G}) - \vec{G}(\vec{\nabla}\cdot\vec{v}). \qquad (23)$$

In the case $\vec{v} = \vec{\Omega}\times\vec{r}$, we have $(\vec{G}\cdot\vec{\nabla})\vec{v} = \vec{\Omega}\times\vec{G}$, and $\vec{\nabla}\cdot\vec{v} = 0$. So (23) may be rewritten as

$$\vec{\nabla}\times(\vec{v}\times\vec{G}) = \vec{\Omega}\times\vec{G} - (v\cdot\vec{\nabla})\vec{G} + \vec{v}(\vec{\nabla}\cdot\vec{G}). \qquad (24)$$

From (24), it follows

$$-(v\cdot\vec{\nabla})\vec{G} + \vec{\Omega}\times\vec{G} = \vec{\nabla}\times(\vec{v}\times\vec{G}) - \vec{v}(\vec{\nabla}\cdot\vec{G}). \qquad (25)$$

After inserting (25) into right-hand side of (22), we get

$$\frac{\partial\vec{G}}{\partial t'} = \frac{\partial\vec{G}}{\partial t} + \vec{\nabla}\times(\vec{v}\times\vec{G}) - \vec{v}(\vec{\nabla}\cdot\vec{G}). \qquad (26)$$

Therefore, the thought for relation between operators $\partial/\partial t'$ and $\partial/\partial t$ is

$$\frac{\partial}{\partial t'} = \frac{\partial}{\partial t} + \vec{\nabla}\times(\vec{v}\times\ ) - \vec{v}(\vec{\nabla}\cdot\ ). \qquad (27)$$

Let us write down relations between vectors $\vec{E}'$, $\vec{B}'$, $\vec{P}'$, $\vec{M}'$, and vectors $\vec{E}$, $\vec{B}$, $\vec{P}$, $\vec{M}$. We adopt the following standard formulas of the special relativity theory:

$$\vec{E}' = \vec{E} - \vec{v}\times\vec{B}, \quad \vec{B}' = \vec{B} + \frac{1}{c^2}\vec{v}\times\vec{E}, \qquad (28)$$

$$\vec{P}' = \vec{P} + \frac{1}{c^2}\vec{v}\times\vec{M}\,, \qquad \vec{M}' = \vec{M} - \vec{v}\times\vec{P}\,. \tag{29}$$

Then, taking into account (12), as a result of substituting (18), (27)–(29) into (9), we obtain the sought for system of Maxwell's equations in a uniformly rotating dielectric medium (to first order in Ω):

$$\begin{aligned}
&\vec{\nabla}\times\vec{E} + \frac{\partial}{\partial t}(\vec{B} + \frac{1}{c^2}\vec{v}\times\vec{E}) = 0\,,\\
&\vec{\nabla}\cdot(\vec{B} + \frac{1}{c^2}\vec{v}\times\vec{E}) = 0\,,\\
&\vec{\nabla}\times\frac{\vec{B}}{\mu_r} - \frac{1}{c^2}\frac{\partial}{\partial t}(\varepsilon_r\,\vec{E} - \vec{v}\times\frac{\vec{B}}{\mu_r}) = 0\,,\\
&\vec{\nabla}\cdot(\varepsilon_r\,\vec{E} - \vec{v}\times\frac{\vec{B}}{\mu_r}) = 0\,.
\end{aligned} \tag{30}$$

System (30) is in agreement (in such approximation) with results of works [27] and [19].

System of wave equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ in a uniformly rotating dielectric medium

Let us rewrite system of Maxwell's equations (30) in other, but equivalent form:

$$\begin{aligned}
&\vec{\nabla}\times\vec{E} + \frac{\partial}{\partial t}(\vec{B} + \frac{1}{c^2}\vec{v}\times\vec{E}) = 0\,,\\
&\vec{\nabla}\cdot(\vec{B} + \frac{1}{c^2}\vec{v}\times\vec{E}) = 0\,,\\
&\vec{\nabla}\times\vec{B} - \frac{1}{c^2}\frac{\partial}{\partial t}(n^2\,\vec{E} - \vec{v}\times\vec{B}) = 0\,,\\
&\vec{\nabla}\cdot(\vec{E} - \frac{1}{n^2}\vec{v}\times\vec{B}) = 0\,.
\end{aligned} \tag{31}$$

Then the system of wave equations for electromagnetic field vectors $\vec{E}$ and $\vec{B}$ in a uniformly rotating dielectric medium may be obtained directly from the system of Maxwell's equations (31). According to results of the author's work [28] [see relations (69) and (70) therein], such system (in a linear with respect to Ω approximation) has the following form:

$$\vec{\nabla}^2\vec{E} - \frac{n^2}{c^2}\frac{\partial^2\vec{E}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\vec{v}\cdot\vec{\nabla})\vec{E} - \vec{\Omega}\times\vec{E}] - \frac{2}{n^2}\vec{\nabla}(\vec{\Omega}\cdot\vec{B}) = 0, \tag{32}$$

$$\vec{\nabla}^2\vec{B} - \frac{n^2}{c^2}\frac{\partial^2\vec{B}}{\partial t^2} + \frac{2}{c^2}\frac{\partial}{\partial t}[(\vec{v}\cdot\vec{\nabla})\,\vec{B} - \vec{\Omega}\times\vec{B}] + \frac{2}{c^2}\vec{\nabla}(\vec{\Omega}\cdot\vec{E}) = 0.$$

Simplified wave equation for transversal components of vectors $\vec{E}$ and $\vec{B}$. Its analytical solutions

Consider the ring laser gyro with a planar N-mirror resonator of arbitrary shape (with perimeter L and area A) which contains along all perimeter a rigid dielectric medium with index of refraction $n = (\varepsilon_r\,\mu_r)^{1/2}$ (the active medium is concentrated only within a infinitely thin layer). In our subsequent calculations, after taking into account Stokes' theorem, it will be convenient to use cylindrical coordinates (ρ, φ, z) and consider such gyro as if it has an equivalent circular resonator of effective radius $\rho = 2A/L$.

The laser gyro is in rest in a uniformly rotating frame of reference $\{\hat{x}\ \hat{y}\ \hat{z}\}$: its resonator plane coincides with the plane $\{\hat{x}\ \hat{y}\}$ of a rotating frame, and its sensitivity axis (which is orthogonal to the resonator plane) coincides with the unit vector $\hat{z}$ of a rotating frame. The device rotates in the inertial space about its sensitivity axis $\hat{z}$ with angular velocity Ω_z, i.e., $\vec{\Omega} = \Omega_z\,\hat{z}$.

The laser gyro resonator provides generation of radiation linearly polarized in the sagittal plane, i.e., vectors $\vec{E}$ and $\hat{z}$ are parallel. The gyro operates at central (of He-Ne active medium emission line) frequency ω_0 (wavelength in vacuum $\lambda_0 = 2\pi c/\omega_0$, wavenumber in vacuum $K_0 = \omega_0/c = 2\pi/\lambda_0$). During the device operation on preselected at initial moment longitudinal mode with very large integer index q, the perimeter stabilization system of the gyro continuously provides (by adjusting parameter ρ) the fulfilment of resonance condition $2\pi\rho = q\lambda$, where $\lambda = \lambda_0/n$ is the wavelength in a dielectric medium with index of refraction n.

For the above-mentioned laser gyro with a circular resonator of radius $\rho = 2A/L$, vectors $\vec{E}$ and $\vec{B}$ may be presented in cylindrical coordinates in the form $\vec{E} = E_z\,\hat{z}$, $\vec{B} = B_\rho\,\hat{\rho}$, where E_z and B_ρ are the transversal components; $\hat{z}$ and $\hat{\rho}$ are the unit vectors. Since we consider the simplest special case of laser gyro rotation about its sensitivity axis $\hat{z}$ when $\vec{\Omega} = \Omega_z\,\hat{z}$, so $\vec{v} = v\hat{\varphi}$, where $v = \rho\Omega_z$, and $\hat{\varphi}$ is the unit vector.

Then, after introducing the more convenient for us longitudinal coordinate $s = \rho\varphi$ ($s = 0, .., 2\pi\rho$), in the approximation of plane waves $\partial\vec{G}/\partial\rho = \partial\vec{G}/\partial z = 0$ ($\vec{G} = \vec{E}, \vec{B}$), and under simplifying condition $2\pi\rho >> \lambda_0$ (see section 4 in [29]) of infinitely small curvature of a circular axis contour of laser gyro resonator (which always fulfils for devices with perimeter more than some centimeters), – we may present the operator $\vec{\nabla}$ in system of wave equations (32) in the form $\vec{\nabla} = \hat{\varphi}(\partial/\partial s)$. After that, the expressions for quantities $\vec{E}$, $\vec{B}$, $\vec{v}$, and $\vec{\nabla}$ must be substituted into equations (32), and their projections onto directions of unit vectors $\hat{z}$ and $\hat{\rho}$ must be taken. As a result, we get:

$$\frac{\partial^2 G}{\partial s^2} - \frac{n^2}{c^2}\frac{\partial^2 G}{\partial t^2} + \frac{2v}{c^2}\frac{\partial^2 G}{\partial s\,\partial t} = 0 \qquad (G = E_z,\, B_\rho). \tag{33}$$

Wave equation (33) must be solved with taking into account the boundary condition $G(s,\, t) = G(s + 2\pi\rho,\, t)$, where $2\pi\rho = q\lambda$, and $\lambda = \lambda_0 / n$.

Method of solving equation (33) for the case of vacuum ($n = 1$) is known from the literature (see, for example, work [20] and formulas (80)–(85) therein).

For our case of a dielectric medium (n > 1), as it follows from (33), expressions for wave transversal components E_z, B_ρ may be constructed (to first order in Ω) as

$$\begin{aligned} E_z(s,\, t) &= E_{z0}\cos(\omega_1 t - K s) + E_{z0}\cos(\omega_2 t + K s), \\ B_\rho(s,\, t) &= B_{\rho 0}\cos(\omega_1 t - K s) - B_{\rho 0}\cos(\omega_2 t + K s), \end{aligned} \tag{34}$$

where

$$\begin{aligned} &B_{\rho 0} = (n/c)E_{z0}, \qquad \omega_1 = (1 - \beta/n)\omega_0, \qquad \omega_2 = (1 + \beta/n)\omega_0, \\ &\beta = v/c = \rho\Omega_z/c, \qquad \rho = 2A/L, \qquad K = nK_0, \qquad K_0 = \omega_0/c = 2\pi/\lambda_0. \end{aligned} \tag{35}$$

Expressions (34) and (35) describe in mathematical form the lows of propagation of transversal electromagnetic waves in a uniformly rotatingresonator of the laser gyro. According to (35), the difference $\Delta\omega = \omega_2 - \omega_1$ between frequencies of counterpropagating in its resonator waves may be calculated by the formula $\Delta\omega = M_g\,\Omega_z$, where

$$M_g = \frac{8\pi A}{\lambda_0 L n} \tag{36}$$

is the geometrical scale multiplier of the gyro. As one can see, expression (36) is in agreement with the known relation (3) for parameter M_g.

Conclusion

As analysis of the literature shows, there are at least six qualitatively different analytical expressions (3)–(8) for geometrical scale multiplier M_g of the ring laser gyro with a planar N-mirror resonator containing along all perimeter L a rigid dielectric medium with index of refraction $n = (\varepsilon_r\,\mu_r)^{1/2}$.

In this work, the expression (3) for parameter M_g is confirmed. In order to do it with a greater level of authenticity, a new approach to derivating the system of Maxwell's equations in a uniformly rotating dielectric medium was used.

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