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SOME FEATURES SATELLITE ATTITUDE CONTROL USING QUATERNIONS

Ua В системах керування орієнтацією сучасних космічних апаратів, насамперед мікросупутників, широкого застосування набули алгоритми з використанням рівнянь кінематики в кватерніонах. Вони забезпечують при поворотах на великі кути уникнення точок сингулярності, характерних для кутів Ейлера-Крилова. Проте специфіка одиничних кватерніонів як параметрів визначення орієнтації може створювати в системі так званий «ефект розкручування», коли поворот космічного апарата в задане положення виконується не найкоротшим шляхом і збільшує витрати електроенергії, запас якої на борту обмежений. Дана публікація, підсумовуючи відомі публікації, ілюструє причини виникнення вказаного ефекту і демонструє простий алгоритм його уникнення. Ефективність такого підходу показана на прикладах чисельного моделювання системи керування мікросупутника з двигунами-маховиками та *PD*-регулятором на платформі *MATLAB*.

En Algorithms using the kinematics equations in quaternions are widely used in the attitude control systems of modern spacecraft, primarily microsatellites. They ensure large-angle rotations and avoidance of singularity points characteristic of Euler-Krylov angles. However, the specificity of single quaternions as attitude representation parameters can create an "unwinding effect" in the control system, when turning the spacecraft to a desirable position is not performed by the shortest path and increases the electricity consumption, that is limited on board. This work, summarizing known publications, illustrates the causes of this effect and demonstrates a simple algorithm for its prevention. The effectiveness of this approach is shown on the MATLAB platform numerical simulation examples of the microsatellite control system with reaction wheels and a PD controller.

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Introduction

The objectives of spacecraft attitude control require considering two important specialties: the possibility of turning to large angles and the restricted supply of energy on board. Due to the first of them, the direct use of traditional Euler-Krylov angles is limited by their singularity points or the “Gimbal lock effect”: when the pitch angle is close to ±90°, the roll and yaw angles coincide. To overcome this problem, for many decades control algorithms based on the spacecraft kinematics equations in quaternions have been used [1] – [3]. The simplest algorithms use a PD controllers based on the quaternion parameters of the deviation from the desired position and their derivatives or the angular rate projections onto the body reference frame [4] - [7].

However, due to the duality of the unit quaternion, the “unwinding effect” may occur, when the rotation of the spacecraft to the desirable position is not performed by the shortest path [8] - [10], which is accompanied by an increase in energy consumption. Therefore, there must be components in the control law that prevent this effect. The main goal of this article is to generalize known publications, to clearly illustrate the causes of the “unwinding effect” and to demonstrate a simple algorithm for its avoidance. The effectiveness of this approach is shown on the MATLAB platform numerical simulation examples of the microsatellite control system with reaction wheels and a PD controller using the following mathematical model.

Statement of the problem

The purpose of this paper is to illustrate the consequences of quaternion duality in the satellite control in the form of “unwinding effect” and to study the satellite control law modernization to avoidance the "unwinding effect".

Duality of the desirable rotation quaternion

Many publications, for example, [9] – [13], are intent to the study of such a feature of quaternions in the spacecraft attitude control, so in this work we will demonstrate the actual mechanism of the problem to clearly show the approach to solution how to prevent this “unwinding effect”.

As is known [2], [3], the quaternion $\boldsymbol{q} = \begin{bmatrix} q_0 & q_1 & q_2 & q_3 \end{bmatrix}$ describes the rotation of a rigid body (or the reference frame associated with it) by a certain angle α around an axis that passes through the center of the rotation circle shown in Fig. 1. The position of this axis in the selected reference frame is determined by the vector part of the quaternion $\boldsymbol{e} = \begin{bmatrix} q_1 & q_2 & q_3 \end{bmatrix}$. Simultaneously the scalar part of the quaternion q_0 depends on the angle α as follows:

$$q_0 = \cos(\alpha/2). \qquad (1)$$

On Fig. 1 it is easy to see that the turn from point A to point B can occur in two ways: a shorter trajectory by an angle $\alpha < \pi$ or a longer one by an angle $\alpha_1 \geq 2\pi - \alpha$. In the second case, for the larger rotation angle α_1, the scalar part of the quaternion q_0' will be as follows:

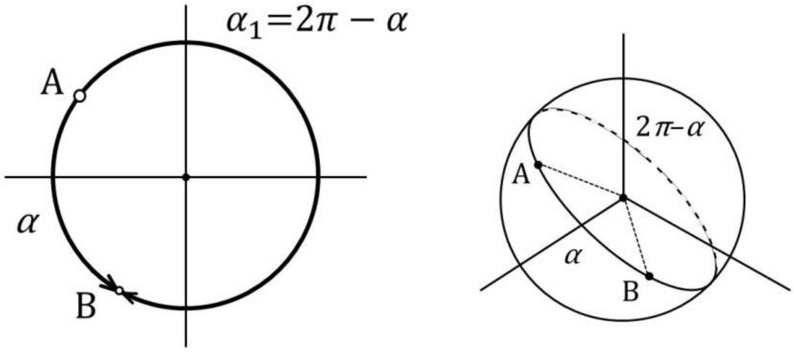


Fig. 1. Two possible rotations between points A and B

$$q_0' = \cos\left(\frac{\alpha_1}{2}\right) = \cos\left(\frac{2\pi - \alpha}{2}\right) = -\cos\left(\frac{\alpha}{2}\right). \qquad (2)$$

Comparing (1) and (2), considering the range of angles, it can be stated that the smaller rotation angle α corresponds to the positive value of the scalar q_0. During the process of spacecraft control, quaternions are calculated using the numerical integration of kinematic equations, vector and matrix operations, because of which the scalar q_0 can have both a positive and a negative value. Therefore, in attitude control algorithms, to ensure its "positivity", as proposed in [4], it is possible to multiply it by the function $\mathrm{sign}(q_0)$:

$\mathrm{sign}(q_0) = 1$ у разі $q_0 > 0$, $\mathrm{sign}(q_0) = -1$ у разі $q_0 < 0$, $\mathrm{sign}(q_0) = 0$ під час $q_0 = 0$.

Mathematical model of spacecraft attitude control system

As an example, consider a common microsatellite attitude control system relative to the satellite orbital reference frame (OSF) $OX_oY_oZ_o$ in Low-Earth orbit. Algorithms for attitude determining such a spacecraft are detailed in [14], and the designations of the reference frames axes and equations of motion used below correspond to [15]. The microsatellite is controlled by three reaction wheels (RW) oriented along the body reference frame (BCF) axes $OX_bY_bZ_b$, and three gyroscopic angular rate sensors (ARS) the same orientation. Then, according to [1], the dynamics equation of the spacecraft angular motion will have the following form (further on, the vectors specified in the projections on the BCF axes are marked in bold with the index "b"):

$$\dot{\boldsymbol{\omega}}_{\boldsymbol{b}} = J_b^{-1}(\boldsymbol{M_b} - \boldsymbol{\omega_b} \times (J_b \boldsymbol{\omega_b}) - \boldsymbol{M_{RW}}), \qquad (3)$$

where "×" is the vector product operation, $\boldsymbol{\omega_b} = \left[\omega_x\, \omega_y\, \omega_z\right]^T$ – the vector of the spacecraft absolute angular rate, J_b – the spacecraft inertia tensor, $\boldsymbol{M_b}$ – the total external disturbing moment vector, $\boldsymbol{M_{RW}}$ is the total mechanical moment vector, generated by three RW with a total angular momentum $\boldsymbol{H_b}$:

$$\boldsymbol{M_{RW}} = \dot{\boldsymbol{H}}_{\boldsymbol{b}} + \boldsymbol{\omega_b} \times \boldsymbol{H_b} \qquad \boldsymbol{M_{RW}} = \dot{\boldsymbol{H}}_{\boldsymbol{b}} + \omega_{\boldsymbol{b}} \times \boldsymbol{H_b}\,. \qquad (4)$$

The quaternion kinematics equations of the spacecraft motion are like [2], [15] and can be written as follows:

$$\dot{\boldsymbol{q}} = \left[W_\times\right] \cdot \boldsymbol{q}^T / 2, \qquad (5)$$

where $\boldsymbol{q} = \left[q_0 \quad q_1 \quad q_2 \quad q_3\right]$is the rotation quaternion of BCF relative to OSF (q_0 is a scalar and $\boldsymbol{e} = \left[q_1 \quad q_2 \quad q_3\right]$is a unit rotation vector), "T" is a matrix transposition symbol, and $[W_\times]$ – skew-symmetric matrix of angular rates:

$$\left[W_\times\right] = \begin{bmatrix} 0 & -(\omega_x - \omega_{xo}) & -(\omega_y - \omega_{yo}) & -(\omega_z - \omega_{zo}) \\ \omega_x - \omega_{xo} & 0 & \omega_z + \omega_{zo} & -(\omega_y + \omega_{yo}) \\ \omega_y - \omega_{yo} & -(\omega_z + \omega_{zo}) & 0 & \omega_x + \omega_{xo} \\ \omega_z - \omega_{zo} & \omega_y + \omega_{yo} & -(\omega_x + \omega_{xo}) & 0 \end{bmatrix} \qquad (6)$$

Here $\boldsymbol{\omega_o} = \left[\omega_{xo} \quad \omega_{yo} \quad \omega_{zo}\right]^T$ is the OSF absolute angular rate vector in projections on its axes. In the case of a circular orbit, without considering its slow precession, the vector $\boldsymbol{\omega_o}$ will be calculate as follows

$$\boldsymbol{\omega_o} = \left[0 \quad 0 \quad -\Omega_{orb}\right]^T, \qquad \Omega_{orb} = \sqrt{\mu / (r_{orb})^3} = \mathrm{const}\,, \qquad (7)$$

wherer_{orb} is the circular orbit radius, μ is the Earth standard gravitational parameter. Then $[W_\times]$ matrix will have the form

$$\left[W_\times^o\right] = \begin{bmatrix} 0 & -\omega_x & -\omega_y & -(\omega_z + \Omega_{orb}) \\ \omega_x & 0 & (\omega_z - \Omega_{orb}) & -\omega_y \\ \omega_y & -(\omega_z - \Omega_{orb}) & 0 & \omega_x \\ (\omega_z + \Omega_{orb}) & \omega_y & -\omega_x & 0 \end{bmatrix} \qquad (8)$$

Without considering the instrumental errors of the on-board equipment, the attitude control law equation of a PD controller with the prevention of the "unwinding effect" (see previous chapter), and with the cross-gyroscopic moments compensation can be represented as follows:

$$\dot{\boldsymbol{H}}_{\boldsymbol{b}} + \boldsymbol{\omega_b} \times \boldsymbol{H_b} = K_P \boldsymbol{e} \cdot \mathrm{sign}(q_0) + K_D \cdot \Delta\boldsymbol{\omega_b} - \boldsymbol{\omega_b} \times \left(J_b \boldsymbol{\omega_b}\right), \qquad (9)$$

where K_P, K_D are the controller parameters (coefficients matrix) for the position and speed deviations, respectively, and

$$\Delta\boldsymbol{\omega}_{\boldsymbol{b}} = \boldsymbol{\omega}_{\boldsymbol{b}} - \boldsymbol{\Omega}_{\boldsymbol{b}}, \qquad (10)$$

Ω_b is the OSF angular rate vector in projections on the BCF axes:

$$\boldsymbol{\Omega}_{\boldsymbol{b}} = \boldsymbol{C}^{\boldsymbol{bo}} \cdot [0 \; -\Omega_o \; 0]^T, \qquad (11)$$

$\boldsymbol{C}^{\boldsymbol{bo}}$ –the direction cosine matrix of the transition from OSF reference frame to BCF

$$\boldsymbol{C}^{\boldsymbol{bo}} = \begin{bmatrix} q_0^2 + q_1^2 - q_2^2 - q_3^2 & 2(q_1q_2 + q_0q_3) & 2(q_1q_3 - q_0q_2) \\ 2(q_1q_2 - q_0q_3) & q_0^2 - q_1^2 + q_2^2 - q_3^2 & 2(q_2q_3 + q_0q_1) \\ 2(q_1q_3 + q_0q_2) & 2(q_2q_3 - q_0q_1) & q_0^2 - q_1^2 - q_2^2 + q_3^2 \end{bmatrix}. \qquad (12)$$

Simulation results of the control algorithm with "unwinding effect" prevention

To illustrate the results of the algorithm of the microsatellite attitude control system, described above in section 2, a simulation model of the system created on the MATLAB platform using equations (3) – (12) is used. Numerical values of system parameters:

– the circular orbit height is 600 km,

– the spacecraft inertia tensor (kg.m^2)

$$J_b = \begin{bmatrix} 0,591 & 0,004 & 0,039 \\ 0,039 & 0,622 & 0,008 \\ 0,008 & 0,004 & 0,613 \end{bmatrix},$$

– the RW maximum mechanical torque (Nm)

$M_{RW}^{max} = [0,004 \;\; 0,004 \;\; 0,004]$, the PD controller parameters, chosen with consideration [13, 16, 17]

$$K_P = \begin{bmatrix} 0,0034 & 0 & 0 \\ 0 & 0,0031 & 0 \\ 0 & 0 & 0,0032 \end{bmatrix}, \quad K_D = \begin{bmatrix} 0,0647 & 0 & 0 \\ 0 & 0,0624 & 0 \\ 0 & 0 & 0,0620 \end{bmatrix}$$

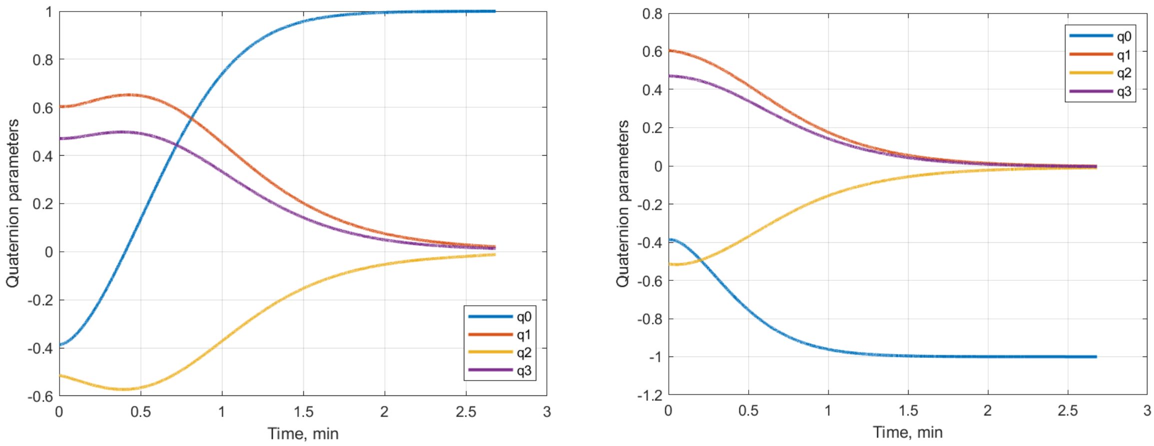
The graphs of changes in the attitude quaternion in the process of rotation the spacecraft by its control system to the OCF axes are shown in Fig. 2 – Fig. 3, illustrate the effectiveness of the algorithm for counteracting the unwinding effect described in chapter 1. These examples correspond to the following two sets of initial values of the Euler angles of the BCF deviation from the OSF reference frame: 1) yaw –80^o, pitch –80^o, roll +175^o and 2) yaw –170^o, pitch –110^o, roll +150^o. The sum of the modules integrals mechanical moments of all three RW on the rotation interval was used as an indicator of energy consumption (marked “Energy” below).

Conclusion

When rotating the spacecraft to large angles, the result of attitude control using quaternions equations depends on the sign of the scalar part of the rotation quaternion. To turn around in the shortest way and, accordingly, with the least energy consumption, it is necessary to ensure the positive sign of the scalar, applying the function signum to its current values.

Control law variants

without anti-unwinding | with use $sign(q_0)$

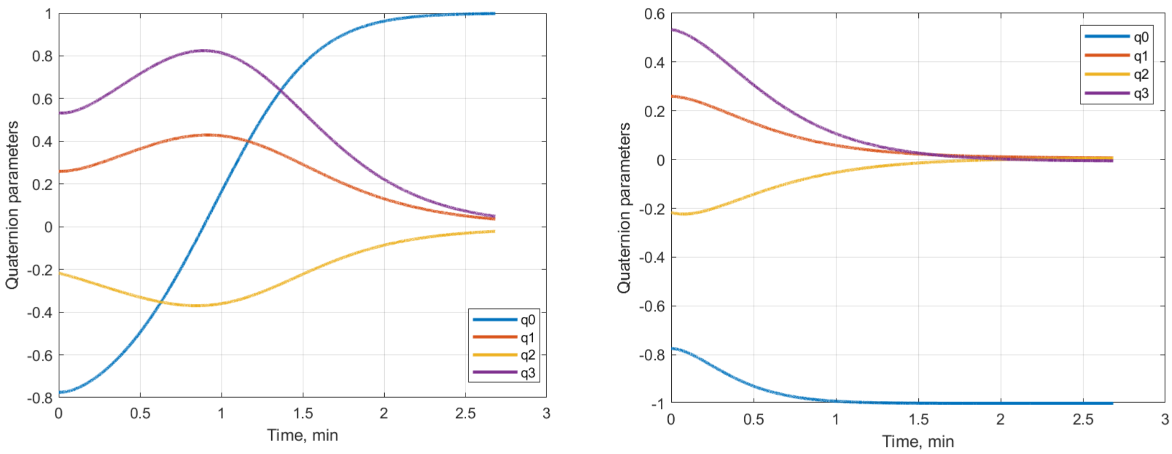


Energy = 0,209 | **Energy = 0,139**

Fig. 2. Simulation results at initial angles yaw –80°, pitch –80°, roll +175°

Control law variants

without anti-unwinding | with use $sign(q_0)$



Energy = 0,205 | **Energy = 0,062**

Рис. 3. Simulation results at initial angles yaw –170°, pitch –110°, roll +150°

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