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ON THE THEORY OF COLLISIONS OF SOLIDS

Ua

У роботі розглянуто випадок зіткнення тіла у формі параболоїда обертання (ударного тіла) і пластини відомої товщини.

Глибину проникнення тіла бойка слід визначати з урахуванням впливу деформації пластини за умови, що обидва тіла, що стикаються, не зруйновані повністю. Наближений розв'язок заснований на застосуванні принципу найменшого примусу (принцип Гаусса).

En

The paper considers the case of collision of a body in the form of a paraboloid of revolution (striker body) and a plate of known thickness.

The depth of penetration of the striker body should be determined, taking into account the influence of plate deformation, provided that both colliding bodies are not completely destroyed.The approximate solution is based on the application of the principle of least coercion (Gauss principle).

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Introduction

The study of shock processes is one of the most pressing problems of applied mechanics associated with the assessment of the behavior of various structures under the influence of intense impulse loads [1, 2, 3].

This problem is becoming more and more important in connection with the emergence of new technological methods of manufacturing various components using collisions, widespread introduction of collision stands and machines testing and research [4].

It should be recognized that the modern theory of collisions of solid objects cannot yet give an answer to numerous questions related to solution of the problem of calculating the endurance of parts of modern high-speed machines, since the physical processes caused by the collision of solid objects are very complex and diverse.

The theory of the collision of solid objects is based on the assumption that the ratio of the initial velocity of collisions to the velocity of propagation of vibrations in the colliding bodies is small, since at high initial velocities of collision there are occurring final deformations and the solution of such problem using the methods of theoretical mechanics and the theory of elasticity may turn out to be untenable.

In this regard, researchers again and again turn to the theory of collisions in order to obtain more complete and accurate solutions that allow deeper and more comprehensive disclosure of the internal laws of the collisions process [5, 6, 7].

Formulation of the problem

The aim of this work is to study the collision of a rigid body and a plate.

In particular, it is a rigid body in the form of a paraboloid of revolution colliding on a plate whose thickness, h, is known.

With a high-speed collision, the rigid body can move into a plastic state, and the plate – into a liquid state.

The depth of penetration, f, of the striker body into the plate needs to be determined. For $f \geq h$, the plate collapses.

Upon collision, the striker body deforms and the magnitude of this deformation is also unknown.

Investigation of the collision process

As is customary in solving many contact problems in the theory of elasticity, we choose a cylindrical coordinate system with the origin at the point of initial contact. The *OZ* axis is the axis of symmetry of the striker body, the shape of which is a paraboloid of revolution (Fig. 1), $\vec{v}_0$ is the velocity at the initial moment of impact.

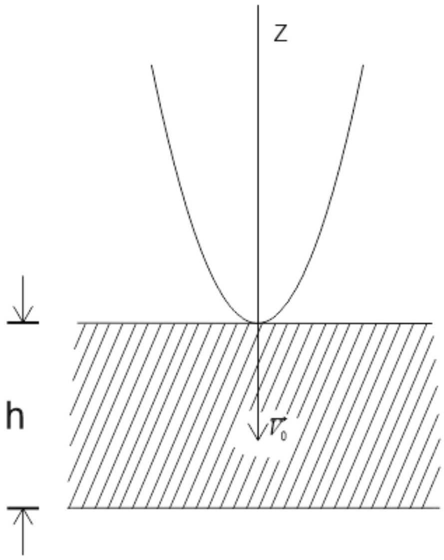

Fig. 1. Paraboloid of revolution Соударение Collision

If the equation of the striker's body shape is

$$z = Ar^2, \tag{1}$$

and the depth of its penetration into the plate is $f(t)$, then in the general case

$$z = -f(t) + A(t)r^2, \tag{2}$$

Here $A(t)$ – is the change in the shape of the striker's body surface (Fig. 2).

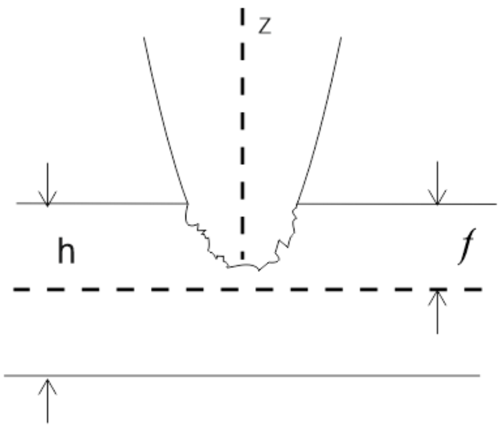

Fig. 2. Local deformation

If the plate does not collapse, then $A(t) = A = \text{const}$.

It is necessary to define $f(t)$. At the same time, $A(t)$, is also unknown.

It is convenient to use the principle of least coercion (Gauss principle) to determine the $f(t)$. The content of the Gauss principle is as follows. For a discrete system of n material points, the compulsion of the system is determined by the equation

$$Z=\frac{1}{2}\sum_{i=1}^{n} m_i \left| \vec{W}_i - \frac{F_i}{m_i} \right|^2 . \qquad (3)$$

Here m_i – are the masses of points, W_i – are their accelerations, F_i – are the active forces applied to the points of the system, as well as the reactions of imperfect connections added to them.

Based on the equations of motion of a non-free system

$$Z=\frac{1}{2}\sum_{i=1}^{n}\frac{R_i^2}{m_i}, \qquad (4)$$

where $\vec{R}_i$ are the reactions of ideal bonds [4].

The Gauss principle states that of all the kinematically possible movements of the points of a non-free system that can occur, the function Z (compulsion) has a minimum for the actual movement of the system.

In monograph [4], by means of simple transformations, the value of Z was obtained for a system consisting of two elastic bodies, which are subjected to contact compression or collision:

$$Z=\iint\limits_{(\omega)} P^2(M,t)\, dS , \qquad (5)$$

where ω – is the contact area, $P(M,t)$ – is the integral average pressure in the contact area.

Thus, the actual movement of the system corresponds to the minimum functional Z. since condition must be satisfied at any moment of time, it (5) can be simplified:

$$Z=\int_0^{\tau} P_m^2(t)\,dt , \qquad (6)$$

here: $P_m(t)$ – is the integral mean pressure found on the interval $Z\left[f(t),0\right]$;

τ – the time of penetration of the striker body into the slab.

For the definition of $P_m(t)$, the kinetic energy change theorem is applied.

Suppose that upon collision, the body (striker) performs only translational motion. Then

$$\frac{mv^2}{2}-\frac{mv_0^2}{2}=Q , \qquad (7)$$

where Q is the work performed by both external and internal forces; $v=\dot{f}$; at $f=0$, $v=v_0$, i.e.

$$\frac{m\dot{f}^2}{2}-\frac{mv_0^2}{2}=Q. \qquad (8)$$

Work Q is performed by forces of various physical nature.

On the body surface in the direction of the OZ axis, two forces act per unit surface of the body:

– force p – hydrodynamic pressure; it creates the so-called pressure resistance;

– force $\frac{\gamma}{g}\cdot\frac{\nu^2}{2}$ – the force that creates high-speed resistance; here γ is the specific gravity of the liquid into which a part of the plate has turned;

$$\nu\cong\nu_z=\frac{dz}{dt}=-\dot{f}+\dot{A}r^2=-\dot{f}+\frac{\dot{A}}{A}(z+f). \qquad (9)$$

The elementary work of these two forces is respectively equal to:

$$dQ_v=\frac{-\gamma\nu^2}{2g}\cos(n,z)\,dsdz, \qquad dQ_p=-p\cos(n,z)\,dsdz, \qquad (10)$$

where $\cos(n,z)$ is the directing cosine of the internal normal to the body surface and the OZ axis.

The total work of the forces of velocity and pressure resistance is

$$Q_v+Q_p=\iiint\limits_V\left(\frac{\gamma\nu^2}{2g}+P\right)dV, \qquad (11)$$

here dV – is an element of the embedded part of the striker body.

The work of the forces of high-speed resistance:

$$Q_v=\iiint\limits_V\frac{\gamma\nu^2}{2g}dV\approx\int_{-f}^{0}k\frac{\gamma\nu^2}{2g}Fdz, \qquad (12)$$

where $F=\pi r^2=\frac{\pi}{A}(z+f)$; from (9) $\nu=-\dot{f}+\frac{\dot{A}}{A}(z+f)$;

k – “shape factor” is determined experimentally.

As a result:

$$Q_v=\frac{\pi k\gamma}{Ag}\left(\frac{1}{4}\frac{\dot{A}^2}{A^2}f^4-\frac{2}{3}\frac{\dot{A}}{A}\dot{f}f^3+\frac{1}{2}\dot{f}^2f^2\right). \qquad (13)$$

Work of forces of pressure resistance:

$$Q_p=\iiint\limits_V pdV. \qquad (14)$$

By the integral mean theorem:

$$Q_p = p_m V_f, \quad \text{where} \quad V_f = \iiint\limits_{(V)} dV = \frac{\pi f^2}{2A}, \tag{15}$$

where V_f – is the volume of the embedded part of the striker body.

Thus:

$$Q_p = P_m \frac{\pi f^2}{2A} \ . \tag{16}$$

In addition to the forces of velocity and pressure resistance, work Q_T is also carried out to change the state of aggregation of the plate substance adjacent to the striker body. The aggregate state of matter changes in the volume of the invading body, i.e.

$$Q_T = æ \iiint\limits_{(V)} \frac{\gamma}{g}\left(C_p T + C_*\right) dV , \tag{17}$$

where $æ$ – mechanical equivalent of heat;

C_p – coefficient of heat capacity of the plate material;

C_* – coefficient of latent heat of fusion;

$T = T_{пл} - T_0$, respectively, the difference between the melting temperature and the initial temperature of the plate.

Total:

$$Q_T = \frac{\pi æ\gamma}{2g}\left(C_p T + C_*\right)\frac{f^2}{A} = \frac{Bf^2}{A} \ , \tag{18}$$

where

$$B = \frac{\pi æ\gamma}{2g}\left(C_p T + C_*\right) . \tag{19}$$

Thus,

$$\frac{m\dot{f}^2}{2} - \frac{mv_0^2}{2} = -Q_P - Q_v - Q_T \ . \tag{20}$$

From here

$$Q_P = \frac{mv_0^2}{2} - \frac{m\dot{f}^2}{2} - \frac{\pi æ\gamma}{2g}\left(\frac{1}{4}\frac{\dot{A}^2}{A^2} f^4 - \frac{2}{3}\frac{\dot{A}}{A}\dot{f}f^3 + \frac{1}{2}\dot{f}^2 f^2\right) - \\ - \frac{f^2}{A}\frac{\pi æ\gamma}{2g}\left(C_p T + C_*\right). \tag{21}$$

Comparing (15) and (21), we obtain

$$P_m = \frac{2A}{\pi f^2}\left[\frac{mv_0^2}{2} - \frac{m\dot{f}^2}{2} - \frac{\pi k\gamma}{2g}\left(\frac{1}{4}\frac{\dot{A}^2}{A^2} f^4 - \frac{2}{3}\frac{\dot{A}}{A}\dot{f}f^3 + \frac{1}{2}\dot{f}^2 f^2\right) - \right.$$
$$\left. - \frac{f^2}{A}\frac{\pi æ\gamma}{2g}\left(C_p \square T + C_*\right)\right]. \tag{22}$$

System enforcement $Z = \int_0^{\tau} P_m^2(t)\,dt$ should be minimal.

If the shape of the striker body changes a little, i.e. $A = \text{const}$, $\dot{A} = 0$, then f_{max} easy to find.

If $f = f_{max}$, then $\dot{f} = 0$, then

$$P_m = \frac{2A}{\pi f^2}\left[\frac{mv_0^2}{2} - \frac{\pi æ\gamma}{2g}\left(C_p T + C_*\right)\frac{f^2}{A}\right] \tag{23}$$

or

$$P_m = \frac{2A}{\pi f^2}\left(\frac{mv_0^2}{2} - \frac{Bf^2}{A}\right) . \tag{24}$$

The minimum Z can be replaced with a minimum P_m^2, since the Gauss principle is a differential principle and Z must have a minimum at any moment, i.e. and at $t = \tau$. The absolute min P_m – is $P_m = 0$, i.e. when speed and pressure resistance disappear. Consequently,

$$f_{\max} = \sqrt{\frac{mv_0^2 A}{2B}} = \sqrt{\frac{mv_0^2 A 2g}{\pi æ\gamma\left(C_P \square T C_*\right)}} . \tag{25}$$

This equality is an approximate estimate f_{max}, since at the moment $t = \tau$ it cannot be $P_m = 0$, i.e. $P_m = 0$ at $t = \tau$, due to relaxation of stresses in colliding bodies.

Findings

The presented method for determining the penetration depth f of the striking body on a plate, the thickness of which h is known, makes it possible to judge the possibility of deformation or complete destruction of the plate (and the striker!) Given known physical characteristics of the substances of both colliding bodies.

The proposed method can be used in the design and use of various types of mechanisms, such as slotting machines, punchers, stamping machines and other mechanisms under the influence of shock loads.

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