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ACCURACY ESTIMATION OF THE CALCULATING ALGORITHM THE SATELLITE'S ORBIT OF EARTH REMOTE SENSING

Ua Розглянуто перетворення систем координат, у яких задаються опорні вектори вихідної навігаційної інформації - потоку сонячного світла і магнітного поля Землі, в систему координат сонячно-синхронної орбіти руху супутника дистанційного зондування Землі матричним і аналітичним способами. Оцінено вплив точності обчислення напрямних косинусів матриць орієнтації супутника на точність визначення його кутів орієнтації та точність сканування. Проведено оцінку впливу на точність визначення орієнтації супутника похибок сенсорів первинної навігаційної інформації та особливостей матричного алгоритму орієнтації. Отримано аналітичні залежності, що дозволяють оцінювати вплив зазначених факторів залежно від положення супутника на орбіті та руху Землі по екліптиці.

En Transformations of the coordinate systems in which the reference vectors of the initial navigation information - the flow of sunlight and the Earth's magnetic field are set - into the coordinate system of the sun-synchronous orbit of the Earth's remote sensing satellite by matrix and analytical methods are considered. The influence of the accuracy of calculating the direction cosines of the satellite

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orientation matrices on the accuracy of determining its orientation angles and the scanning accuracy is estimated. The estimation of the influence on the accuracy of determining the satellite orientation of the sensor errors of the primary navigation information and the features of the matrix orientation algorithm is carried out. Analytical dependencies are obtained that allow one to evaluate the influence of the noted factors depending on the position of the satellite in its orbit and the motion of the Earth along the ecliptic.

Introduction

Small spacecraft are increasingly used due to the reduction of energy and mass characteristics of onboard equipment, reducing the time of development and cost of launch services, the possibility of using a light launch vehicle. At the same time, the requirements for the quality and accuracy of operation in orbit are growing with limited possibilities of using high-precision but dimensional on-board systems. This determines the relevance of a more thorough study of algorithms that use information from primary sensors of low accuracy. Approaches to the development of the system of orientation of microsatellites are described in [1 - 3]. A three-axis magnetometer and solar sensors are used. Algorithms for calculating orientation quaternions based on solar sensor signals are considered in [4 - 8]. Modern algorithms for the active magnetic orientation of satellites have been studied in [9]. Errors in determining the orientation by matrix algorithms for the case of small angles are considered [10]. They show good smoothing of angle estimates without the use of low-frequency filtering of signals from primary sensors [11].

It remains important to study the sensitivity of orientation algorithms to implementation errors due to the finiteness of the processor bit grid.

Formulation of the problem

Let us determine the influence of errors in setting (calculating the position) of the orbit of the Earth remote sensing satellite (ERS) on the accuracy of the orientation of the satellite scanner axis.

The solution of the problem

The accuracy of the position of the axis of the optical scanner of the ERS satellite in relation to the Earth is determined by the accuracy of calculating the position of the satellite's orbit in the earth's coordinate system and the accuracy of determining the orientation of the satellite relative to its orbit under the assumption that the scanner is rigidly connected to the satellite. The solution of the second part of the problem in the case of small satellites in a sun-synchronous orbit is often performed using data on the parameters of the Earth's magnetic field, obtained by magnetic sensors, and the parameters of the radius-vector of the Sun, formed by the sensors of the Sun.

Estimation of the accuracy of calculating the position of the satellite orbit in the earth's coordinate system

To solve the problem of calculating the position of the satellite orbit, we introduce the following coordinate systems (Fig. 1):

1. Inertial (ICS) $C\xi\eta\zeta$ centered at the center of mass of the Sun, the ξ and η axes lie in the plane of the ecliptic (the η axis is directed towards the North Star), the ζ axis is directed along the normal to the ecliptic. The position of the Earth's center of mass on the ecliptic is set by the angle φ_3, which forms the radius vector of the Earth's center of mass with the ξ axis.
2. Geocentric (GCS) $З\xi_i\eta_i\zeta_i$ (non-inertial), the axes of which are parallel to the corresponding axes of the ISC $C\xi\eta\zeta$.
3. Geocentric solar (GSCS) $З\xi_{3C}\eta_{3C}\zeta_{3C}$, in which the ξ_{3C} axis is directed along the radius vector of the Earth's center of mass from the Sun, η_{3C} is tangential to the Earth's orbit in the direction of its movement along the ecliptic; the angle $\xi_{3C}\,{}^\wedge\xi_i$ is equal to φ_3.

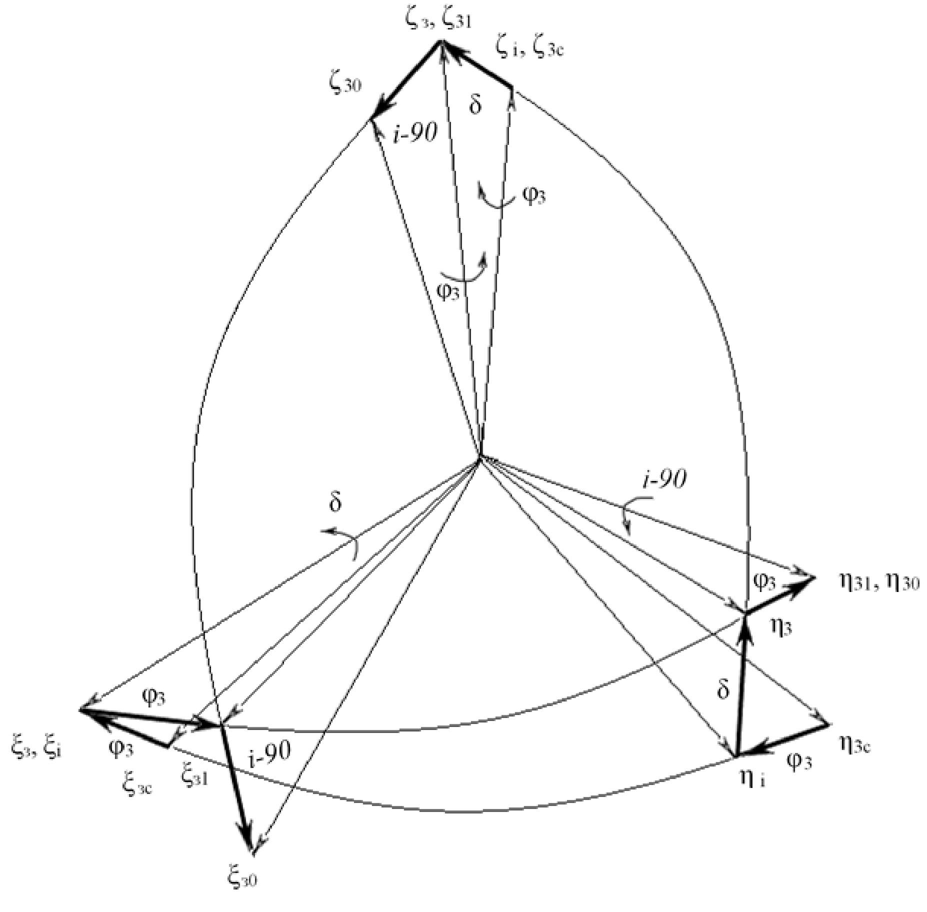

Fig. 1. Coordinates systems

4. Geocentric terrestrial (GTCS) 3ξ3η3ζ3, the axis of which (the axis of rotation of the Earth) is directed to the polar star and is inclined to the ecliptic at an angle of 900 - δ.
5. Terrestrial solar (TSCS) 3ξ31η31ζ31. The η313ζ31 plane is located towards the Sun, the ξ313η31 plane is the plane of the Earth's equator.
6. Orbital terrestrial 3ξ3Oη3Oζ3O. The plane of the orbit η3O3ζ3O is inclined to the equatorial plane at an angle i around the axis 3η3O. The 3ζ3O axis forms an angle i - 900 with the 3ζ31 axis.
7. Orbital satellite SxOyOzO. yOSzO is the plane of the sun-synchronous orbit, the SzO axis is directed tangentially to the orbit along the satellite's velocity vector, SyO is directed along the satellite's center of mass radius- vector from the Earth's center of mass. The position of the satellite in orbit is determined by the arc coordinate φS from the plane of the earth's equator (3η3O axis).
8. Satellite linked coordinate system SxSySzS, the axes of which are invariably linked to the satellite.

The connection of coordinate systems is shown by the following transformation diagram:

$$\xi_{3C}\eta_{3C}\zeta_{3C}\xrightarrow[\zeta_{3C}\zeta_i]{-\varphi_3}\xi_i\eta_i\zeta_i\xrightarrow[\xi_i\xi_3]{\delta}\xi_3\eta_3\xi_3\xrightarrow[\zeta_3\zeta_{31}]{\varphi_3}$$
$$\xrightarrow[\zeta_3\zeta_{31}]{\varphi_3}\xi_{31}\eta_{31}\zeta_{3C}\xrightarrow[\eta_{31}\eta_{30}]{i-90^\circ}\xi_{30}\eta_{30}\zeta_{30}\xrightarrow[\xi_{30}x_0]{\varphi_S}x_0y_0z_0\ . \tag{1}$$

The square transformation matrices of coordinate systems by rotations around the indicated axes by angles according to diagram (1) are denoted $A_0(-\varphi_3)$, $A_1(\delta)$, $A_2(\varphi_3)$, $A_3(i\text{-}90^0)$, $A_4(\varphi_s)$. Matrix expressions can be easily obtained from the direction cosines of the unit vectors of the new coordinate system in the original coordinate system. For example,

$$A_0(-\varphi_3)=\begin{matrix}\cos\varphi_3 & \sin\varphi_3 & 0\\ -\sin\varphi_3 & \cos\varphi_3 & 0\\ 0 & 0 & 1\end{matrix}\ . \tag{2}$$

For orientation of small satellites, information about the Earth's magnetic field and the flow of sunlight at the location of the satellite in orbit is used, among other things. Therefore, it is necessary to know the mutual orientation of the orbital earth coordinate system $\xi_{3O}\eta_{3O}\zeta_{3O}$ (the plane of the satellite orbit) and the geocentric solar coordinate system $\xi_{3C}\eta_{3C}\zeta_{3C}$, as well as the orbital satellite coordinate system $x_Oy_Oz_O$ and the terrestrial solar coordinate system $\xi_{31}\eta_{31}\zeta_{31}$:

$$[\xi_{3O}\eta_{3O}\zeta_{3O}]^T = A_3^T A_2^T A_1^T A_0^T\ [\xi_{3C}\eta_{3C}\zeta_{3C}]^T, \tag{3}$$

$$[x_Oy_Oz_O]^T = A_4^T A_3^T[\xi_{31}\eta_{31}\zeta_{31}]^T. \tag{4}$$

Orientation matrices of coordinate systems can be easily determined by the basic formula of spherical trigonometry, which, for example, for a spherical triangle $\xi_{3O}\ \eta_i\ \zeta_3$ (Fig. 1) has the form

$$\cos(\xi_{3O}|\eta_i) = \cos(\xi_{3o}|\zeta_3)\cos(\zeta_3|\eta_i) + \\ + \sin(\xi_{3o}|\zeta_3)\sin(\zeta_3|\eta_i)\cos(90^0 - \varphi_3). \quad (5)$$

Let's find the coordinate transformation matrix (3), which determines the projection of the Sun vector (sunlight) on the axis of the orbital earth coordinate system:

$$\mathrm{A}_5 = \mathrm{A}_4^{\mathrm{T}}\mathrm{A}_3^{\mathrm{T}}\mathrm{A}_2^{\mathrm{T}}\mathrm{A}_1^{\mathrm{T}}\mathrm{A}_0^{\mathrm{T}} = \begin{bmatrix} a_{11} & \cdots & a_{13} \\ \vdots & \ddots & \vdots \\ a_{31} & \cdots & a_{33} \end{bmatrix}. \quad (6)$$

Here there are

$$\begin{aligned} a_{11} &= \cos^2\varphi_3\cos(i-90°) + \sin\varphi_3[\sin\delta\sin(i-90°) + \\ &\quad + \cos\delta\cos(i-90°)\sin\varphi_3]; \\ a_{21} &= c_{21}\cos\varphi_s + c_{31}\sin\varphi_s; \\ a_{31} &= -c_{21}\sin\varphi_s + c_{31}\cos\varphi_s; \\ c_{21} &= (\cos\delta - 1)\sin\varphi_3\cos\varphi_3, \\ c_{31} &= \cos^2\varphi_3\sin(i-90^0) + \\ &\quad + \sin\varphi_3[\sin(i-90^0)\sin\varphi_3\cos\delta - \cos(i-90^0)\sin\delta]. \end{aligned} \quad (7)$$

The matrix, which determines the projection of the vector of the strength of the Earth's magnetic field (terrestrial solar coordinate system) on the axis of the orbital satellite coordinate system (Fig. 2), will be

$$A_6 = A_4^T A_3^T = \begin{bmatrix} b_{11} & \cdots & b_{13} \\ \vdots & \ddots & \vdots \\ b_{31} & \cdots & b_{33} \end{bmatrix}, \quad (8)$$

where

$$\begin{aligned} b_{11} &= \cos(i-90^0); \quad b_{31} = \cos\varphi_3\sin(i-90^0), \\ b_{23} &= \sin\varphi_S\cos(i-90^0), \quad b_{33} = \cos\varphi_S\cos(i-90^0). \end{aligned} \quad (9)$$

Matrices (6) and (8) determine the luminous flux (vector of the Sun) and the magnetic field in the satellite orbital coordinate system, which are taken as known reference vectors for calculating the satellite orientation cavternion [10].

The error in calculating the direction cosines of the matrices (6), (8), which may be due to the limitation of the bit width of the calculations, and the corresponding angles of orientation of the reference vectors, directly enters into the error in the orientation of the satellite relative to the Earth. Let the actual value of the orientation angle α be determined with an error Δ_α: $\alpha = \bar{\alpha} + \Delta_\alpha$, where $\bar{\alpha}$ is the estimate of the angle.

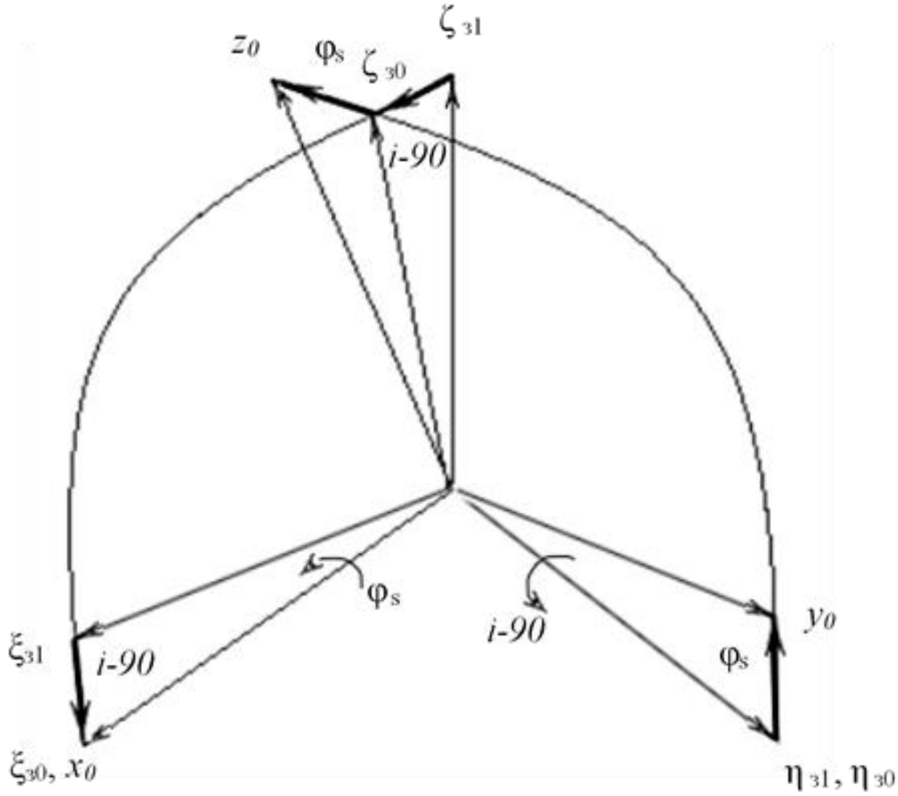

Fig. 2. Coordinates systems

Let's find the angle error Δ_α, caused by an approximate calculation of the direction cosine of this angle in the orientation matrices (6), (8). Assuming the angle error is small ($\Delta_\alpha \ll 1$), we get

$$\cos(\bar{\alpha}+\Delta_\alpha)=\cos\bar{\alpha}(1\text{-}0{,}5\Delta_\alpha^2)\text{-}\Delta_\alpha\sin\bar{\alpha}=$$

$$=\cos\bar{\alpha}-\left(\Delta_\alpha\sin\bar{\alpha}+0{,}5\Delta_\alpha^2\cos\bar{\alpha}\right)=\bar{C}+\Delta_C;$$

$$\bar{C}=\cos\bar{\alpha}; \qquad |\Delta_C|=0{,}5\Delta_\alpha^2\cos\bar{\alpha}+\Delta_\alpha\sin\bar{\alpha} ,$$

whence for small Δ_α and final $\bar{\alpha}$ will find

$$\Delta_\alpha=\left|\frac{\Delta_C}{\sin\bar{\alpha}}\right| , \tag{10}$$

and for values $\bar{\alpha}\to 0$ and small $\bar{\alpha}$

$$\Delta_\alpha=\sqrt{2|\Delta_C|}. \tag{11}$$

As follows from (10), the error in calculating the angle, the value of which enters the satellite attitude control system, depends on the value of the angle itself (it is multiplicative). An analysis of the elements of the direction cosine matrices (2), (7), (9) shows the presence of angles in them varying in the range (0^0 - 360^0) when the Earth moves along the ecliptic and the satellite moves along the orbit, which can cause cases of small values of the angles of orientation of the satellite and an increase in the computational error (10). The table shows numerical estimates of the errors in determining the orientation angles, depending on the value of the small parts of the direction cosines discarded in the calculations, and caused by this the error $\Delta l = \mathrm{H}\sin\Delta_\alpha$ of the image determination by the ERS scanner for several values of $\bar{\alpha}$ and the height H = 600 km of the orbit.

***Table*.**

Error estimates

	$\bar{\alpha}=30^0$		$\bar{\alpha}=0^0$	
Δ_C	Δ_α, deg/rad	Δ_ℓ, km	Δ_α, deg/rad	Δ_ℓ, km
$1\cdot10^{-5}$	$0{,}001^0/2\cdot10^{-5}$	0,012	$0{,}2^0/0{,}0033$	2
$1\cdot10^{-4}$	$0{,}01^0/2\cdot10^{-4}$	0,12	$0{,}8^0/0{,}014$	8,4
$1\cdot10^{-3}$	$0{,}1^0/2\cdot10^{-3}$	1,2	$2{,}6^0/0{,}045$	27
$4{,}4\cdot10^{-3}$	$0{,}5^0/8{,}8\cdot10^{-3}$	5,6	$5{,}1^0/0{,}09$	54

Expressions (10), (11) and numerical values (Table) allow you to determine the required processor capacity to ensure the required orientation accuracy of the remote sensing scanner.

Conclusions

The obtained relations make it possible to compose transformation matrices of coordinate systems depending on the position of the Earth's remote sensing satellite orbit and the space in which the parameters of the navigation field are determined, measured by the onboard sensors of primary information. The dependence of the accuracy of satellite orientation relative to the Earth on the accuracy of calculating the elements of the orientation matrix (orientation quaternion) of the satellite is determined. It is shown that the above dependences allow to determine the required digit capacity of the computing facilities to ensure the required accuracy of the Earth remote sensing system.

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