MECHANICAL CHARACTERISTICS DETERMINATION OF AIRCRAFT WING CARBON PLASTIC PANEL

The paper presents the method of calculating the mechanical characteristics of multilayer composite panels. The optimal sequence of laying out monolayers was selected. The mechanical behavior of the composition is determined by the ratio of the properties of the reinforcing elements and the matrix, as well as the strength of the connection between them. Among the most important requirements for the designs of modern aircraft, we can mention the minimum weight, the maximum stiffness and strength of the nodes, the maximum service life of the structures in operational conditions, and high reliability. With the help of the developed methodology, the elastic properties of the composite material with different angles of laying layers were determined. Calculations of the mechanical characteristics of carbon-plastic panels made of carbon tape and carbon fabric are given.

Introduction

In connection with the wide use of various composite materials, especially in aircraft construction, there are very relevant and important tasks of developing methods for assessing the strength of composite materials, creating mathematical models of deformation, developing methods for experimental research of deformation and strength properties of structural composite materials, as well as evaluating dangers of technological and operational defects arising in structural elements.

Composite materials bring a lot of useful things to aviation - they increase the strength of parts, reduce their weight and susceptibility to corrosion, and also

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allow to reduce the number of parts. It is known that a composite material consists of a high-strength filler oriented in a certain direction and a matrix.

A variety of fibers and matrix materials, as well as reinforcement schemes, used in the creation of composite structures, which allows us to purposefully adjust the strength, stiffness, level of operating temperatures and other properties by selecting the composition, changing the ratio of components and the microstructure of the composite [1 - 3, 7].

High-modulus carbon fibers are used for the manufacture of aircraft parts. Polymeric carbon plastics are characterized by low density, high modulus of elasticity, low thermal and electrical conductivity, low frictional wear and high damping capacity. By orienting the fibers at an angle to each other, it is possible to change the damping capacity of carbon fiber plastics to a large extent and to rebuild parts from the resonance mode without changing their geometric shapes [41 - 6].

Creation of monolayer packages

To identify any monolayer in a package of monolayers, the layer orientation code is used, which defines [3]:
- angle of inclination of the monolayer to the basic axis of the monolayer package X;
- the number of monolayers having a given angle of inclination;
- exact arrangement of monolayers.

Each monolayer is marked with a number showing the orientation of the monolayer in degrees between the direction of its fibers and the X axis. Fig. 1 shows the standard orientation of monolayers 0°, +45°, -45° i 90°.

Fig. 1. Index of standard orientation of monolayers
Adjacent monolayers are divided by a diagonal line if their tilt angles are different. Monolayers are written sequentially from the front surface of one monolayer to another using parentheses.

Adjacent monolayers having the same angle are indicated by a numerical subscript. The subscript "T" in parentheses indicates that the complete set of monolayers is given. If adjacent monolayers have the same but opposite angle, the corresponding "+" or "-" signs are used. Counter-clockwise angles are considered positive.

Sometimes, instead of the negative angles of the first quadrant, the positive angles found in the second quadrant are used. For example, instead of the designation of the angle $-45^\circ$, the notation is used $135^\circ$.

**Dependence of stresses and strains for different coordinate systems**

For a plane stress state when the axes are rotated by an angle $\varphi^\circ$ (fig. 2), the dependence of the acting stresses in the coordinate systems of the monolayer and the package of monolayers has the form

$$
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} = 
\begin{bmatrix}
c^2 & s^2 & -2 \cdot s \cdot c \\
2 \cdot s \cdot c & c^2 & s^2 \\
-2 \cdot s \cdot c & s^2 & c^2 - s^2
\end{bmatrix} \cdot 
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix},
$$

$$
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} = 
\begin{bmatrix}
c^2 & s^2 & 2 \cdot s \cdot c \\
2 \cdot s \cdot c & c^2 & -2 \cdot s \cdot c \\
-s \cdot c & s \cdot c & c^2 - s^2
\end{bmatrix} \cdot 
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix},
$$

where $c = \cos \varphi^\circ$; $s = \sin \varphi^\circ$,

$\sigma_1$, $\sigma_2$, $\tau_{12}$ – are the stresses acting in the monolayer,

$\sigma_x$, $\sigma_y$, $\tau_{xy}$ – are the stresses acting in the monolayer packets.

Fig. 2. A monolayer rotated by an angle $\varphi^\circ$ with respect to the coordinate system of the monolayer package
The dependence of deformations in the coordinate systems of a monolayer and a package of monolayers has the form:
\[
\begin{align*}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix} &= \begin{bmatrix}
c^2 & s^2 & -sc \\
s^2 & c^2 & sc \\
2sc & -2sc & c^2 - s^2
\end{bmatrix} \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix}, \\
\begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\gamma_{12}
\end{bmatrix} &= \begin{bmatrix}
c^2 & s^2 & sc \\
s^2 & c^2 & -sc \\
-2sc & 2sc & c^2 - s^2
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix},
\end{align*}
\]
where $\varepsilon_1, \varepsilon_2, \gamma_{12}$ – are the monolayer deformations,
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$ – are the package of monolayers deformations.

**Calculation of elastic characteristics of composite multilayer material**

Each individual layer (monolayer) consists of unidirectional fibers that determine the direction of the layer, and a matrix that provides normal and transverse stiffness of the layer. Such a monolayer is orthotropic because it has two mutual axes of symmetry. Its characteristic feature is that normal stresses acting along the axes of orthotropy do not cause shear deformations, and tangential stresses - elongations. Hooke's law describing the stress-strain relationship for a unidirectional monolayer in a flat stress-strain state has the form:
\[
\begin{align*}
\begin{bmatrix}
\sigma_1 \\
\sigma_2 \\
\tau_{12}
\end{bmatrix} &= \begin{bmatrix}
C_{11}^0 & C_{12}^0 & 0 \\
C_{21}^0 & C_{22}^0 & 0 \\
0 & 0 & C_{66}^0
\end{bmatrix} \begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix},
\end{align*}
\]
where $\sigma_1, \sigma_2, \tau_{12}$ – are the stresses acting in the monolayer;
$\varepsilon_1, \varepsilon_2, \gamma_{12}$ – are the monolayer deformations;
$C_{kl}^0$ – are the coefficients of the stiffness matrix of the monolayer, which are determined as:
\[
C_{11}^0 = \frac{E_1}{1 - \mu_{12} \cdot \mu_{21}}; \\
C_{12}^0 = \frac{E_1 \cdot \mu_{21}}{1 - \mu_{12} \cdot \mu_{21}} = \frac{E_2 \cdot \mu_{12}}{1 - \mu_{12} \cdot \mu_{21}}; \\
C_{22}^0 = \frac{E_2}{1 - \mu_{12} \cdot \mu_{21}}; \\
C_{66}^0 = G_{12},
\]
where $E_1, E_2$ are the longitudinal and transverse modulus of elasticity of the monolayer;
$G_{12}$ is the monolayer shear modulus;
$\mu_{12}$ is the Poisson’s main ratio;


\[ \mu_{21} \text{ is the second-order Poisson's ratio, which is determined from Maxwell's relation:} \]
\[ \mu_{12} \cdot E_2 = \mu_{21} \cdot E_{12}. \]

Typical elastic characteristics of monolayers of carbon tape and carbon fabric are presented in Tab. 1.

**Table 1**

Elastic characteristics of the monolayer

<table>
<thead>
<tr>
<th>monolayer</th>
<th>Moduli of elasticity and shear, MPa</th>
<th>Poisson's ratios</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_1 )</td>
<td>( E_2 )</td>
</tr>
<tr>
<td>carbon tape</td>
<td>143000</td>
<td>8400</td>
</tr>
<tr>
<td>carbon fabric</td>
<td>65000</td>
<td>63000</td>
</tr>
</tbody>
</table>

If the loading of the monolayer does not occur along the orientation axis, then it is in the state of layer-by-layer loading as part of the composite package. Then Hooke’s law takes shape:

\[
\begin{bmatrix}
\sigma_x \\
\sigma_y \\
\tau_{xy}
\end{bmatrix} =
\begin{bmatrix}
C_{11}^0 & C_{12}^0 & C_{16}^0 \\
C_{12}^0 & C_{22}^0 & C_{26}^0 \\
C_{16}^0 & C_{26}^0 & C_{66}^0
\end{bmatrix}
\begin{bmatrix}
\varepsilon_x \\
\varepsilon_y \\
\gamma_{xy}
\end{bmatrix},
\]

where the coefficients of the stiffness matrix of the monolayer rotated by an angle \( \varphi^o \)

\[
C_{11}^0 = V_1 + V_2 \cdot \cos 2\varphi + V_3 \cdot \cos 4\varphi;
\]
\[
C_{12}^0 = V_1 - 2 \cdot V_4 - V_3 \cdot \cos 4\varphi;
\]
\[
C_{16}^0 = 0,5 \cdot V_2 \cdot \sin 2\varphi + V_3 \cdot \sin 4\varphi;
\]
\[
C_{22}^0 = V_1 - V_2 \cdot \cos 2\varphi + V_3 \cdot \cos 4\varphi;
\]
\[
C_{26}^0 = 0,5 \cdot V_2 \cdot \sin 2\varphi - V_3 \cdot \sin 4\varphi;
\]
\[
C_{66}^0 = V_4 - V_3 \cdot \cos 4\varphi.
\]

Here, the independent coefficients \( V_1, V_2, V_3 \) and \( V_4 \) are determined:

\[
V_1 = \left( 3 \cdot C_{11}^0 + 2 \cdot C_{12}^0 + 3 \cdot C_{22}^0 + 4 \cdot C_{66}^0 \right) / 8;
\]
\[
V_2 = \left( C_{11}^0 - C_{22}^0 \right) / 2;
\]
\[
V_3 = \left( C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 - 4 \cdot C_{66}^0 \right) / 8;
\]
\[
V_4 = \left( C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 + 4 \cdot C_{66}^0 \right) / 8.
\]

Coefficients \( V_1 \) and \( V_4 \) characterize the average stiffness of the monolayer under tension and shear, and coefficients \( V_2 \) and \( V_3 \) characterize the degree of
anisotropy of the material. Thus, the behavior of a monolayer in a flat stress-strain state is characterized by four independent elastic constants: $E_1$, $E_2$, $G_{12}$, $\mu_{12}$ for reinforcement angles $0^\circ$ i $90^\circ$; $V_1$, $V_2$, $V_3$, $V_4$ for reinforcement angles $\phi^\circ$.

Elastic characteristics of a monolayer rotated by an angle $\phi^\circ$:

$$E_x = \frac{\Delta C}{C_{22}^\phi \cdot C_{66}^\phi - \left(C_{26}^\phi\right)^2}; \quad G_{xy} = \frac{\Delta C}{C_{11}^\phi \cdot C_{22}^\phi - \left(C_{12}^\phi\right)^2};$$

$$E_y = \frac{\Delta C}{C_{11}^\phi \cdot C_{66}^\phi - \left(C_{16}^\phi\right)^2}; \quad \mu_{xy} = \frac{\Delta C}{C_{12}^\phi \cdot C_{66}^\phi - C_{16}^\phi \cdot C_{26}^\phi \left(C_{26}^\phi\right)^2},$$

where $\Delta C$ is the determinant of the stiffness matrix

$$\Delta C = \det \begin{bmatrix} C_{11}^\phi & C_{12}^\phi & C_{16}^\phi \\ C_{12}^\phi & C_{22}^\phi & C_{26}^\phi \\ C_{16}^\phi & C_{26}^\phi & C_{66}^\phi \end{bmatrix}.$$

Changes in the modulus of elasticity and shear of the carbon tape (Tab. 1) depending on the angle $\phi^\circ$ are presented in fig. 3, and Poisson’s ratios are presented in fig. 4. Changes in modulus of elasticity and shear of carbon fabric (Tab. 1) depending on the angle $\phi^\circ$ are presented in fig. 5, and Poisson’s coefficients in fig. 6.
Fig. 5. The modulus of elasticity and shear of carbon fiber depending on the angle $\varphi^\circ$

Fig. 6. Poisson’s ratios of carbon fiber depending on the angle $\varphi^\circ$

Determination of elastic characteristics of carbon fiber for all angles $\varphi$

Monolayer stiffness matrix coefficients

$$C_{11}^0 = \frac{E_1}{1 - \mu_{12} \cdot \mu_{21}} = \frac{65000}{1 - 0.07 \cdot 0.06785} = 65310 \text{ (MPa)};$$

$$C_{12}^0 = \frac{E_1 \cdot \mu_{21}}{1 - \mu_{12} \cdot \mu_{21}} = \mu_{21} \cdot C_{11}^0 = 0.06785 \cdot 65310 = 4431 \text{ (MPa)};$$

$$C_{22}^0 = \frac{E_2}{1 - \mu_{12} \cdot \mu_{21}} = \frac{63000}{1 - 0.07 \cdot 0.06785} = 63301 \text{ (MPa)};$$

$$C_{66}^0 = C_{12} = 6500 \text{ (MPa)}$$

Independent coefficients are

$$V_1 = \left(3 \cdot C_{11}^0 + 2 \cdot C_{12}^0 + 3 \cdot C_{22}^0 + 4 \cdot C_{66}^0\right) / 8 =$$

$$= \left(3 \cdot 65310 + 2 \cdot 4431 + 3 \cdot 63301 + 4 \cdot 6500\right) / 8 = 52587 \text{ (MPa)};$$

$$V_2 = \left(C_{11}^0 - C_{22}^0\right) / 2 = \left(65310 - 63301\right) / 2 = 1005 \text{ (MPa)};$$

$$V_3 = \left(C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 - 4 \cdot C_{66}^0\right) / 8 =$$

$$= \left(65310 - 2 \cdot 4431 + 63301 - 4 \cdot 6500\right) / 8 = 11719 \text{ (MPa)};$$

$$V_4 = \left(C_{11}^0 - 2 \cdot C_{12}^0 + C_{22}^0 + 4 \cdot C_{66}^0\right) / 8 =$$

$$= \left(65310 - 2 \cdot 4431 + 63301 + 4 \cdot 6500\right) / 8 = 18219 \text{ (MPa)}.$$

Coefficients of the stiffness matrix of a monolayer rotated at an angle $\varphi^\circ = 45^\circ$
Determine the stiffness matrix is

\[
\Delta C = \begin{vmatrix}
  C_{11} & C_{12} & C_{16} \\
  C_{12} & C_{22} & C_{26} \\
  C_{16} & C_{26} & C_{66}
\end{vmatrix} = \begin{vmatrix}
  40868 & 27868 & 502 \\
  27868 & 40868 & 502 \\
  502 & 502 & 29937
\end{vmatrix} = 2,67445 \cdot 10^{13} (\text{MPa})^3.
\]

Elastic characteristics of a monolayer turned at an angle \( \varphi = 45^\circ \)

\[
E_x = \frac{\Delta C}{C_{22} \cdot C_{66} - (C_{26})^2} = \frac{2,67445 \cdot 10^{13}}{40868 \cdot 29937 - 502^2} = 21864 \text{ (MPa)};
\]

\[
E_y = \frac{\Delta C}{C_{11} \cdot C_{66} - (C_{16})^2} = \frac{2,67445 \cdot 10^{13}}{40868 \cdot 29937 - 502^2} = 21864 \text{ (MPa)};
\]

\[
G_{xy} = \frac{\Delta C}{C_{11} \cdot C_{22} - (C_{12})^2} = \frac{2,67445 \cdot 10^{13}}{40868 \cdot 40868 - 27868^2} = 29930 \text{ (MPa)};
\]

\[
\mu_{xy} = \frac{\Delta C}{C_{12} \cdot C_{66} - (C_{16})^2} = \frac{2,67445 \cdot 10^{13}}{40868 \cdot 29937 - 502^2 \cdot 502} = 0,68.
\]

Similarly, the elastic characteristics of a monolayer rotated at other angles \( \varphi \) are determined. The values of the modulus of elasticity and shear depending on the angle \( \varphi \) in the polar coordinate system are presented in fig. 7, and Poisson's ratio in fig. 8.

For typical reinforcement angles, the values of elastic characteristics of a monolayer of carbon fiber in the coordinate system of a package of monolayers are presented in Tab. 2.
Fig. 7. The modulus of elasticity $E$ and shear $G$ for carbon fiber depending on the angle $\phi^\circ$ in the polar coordinate system

Fig. 8. Poisson’s ratio for carbon fiber depending on the angle $\phi^\circ$ in the polar coordinate system
Change in the elastic characteristics of a carbon fiber monolayer in the coordinate system of a package of monolayers

<table>
<thead>
<tr>
<th>φ</th>
<th>$E_x$ (МПа)</th>
<th>$E_y$ (МПа)</th>
<th>$G_{xy}$ (МПа)</th>
<th>$\mu_{xy}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>65000</td>
<td>63000</td>
<td>6500</td>
<td>0,070</td>
</tr>
<tr>
<td>15</td>
<td>43583</td>
<td>42795</td>
<td>8082</td>
<td>0,375</td>
</tr>
<tr>
<td>30</td>
<td>26255</td>
<td>26088</td>
<td>15743</td>
<td>0,621</td>
</tr>
<tr>
<td>45</td>
<td>21864</td>
<td>21864</td>
<td>29930</td>
<td>0,682</td>
</tr>
<tr>
<td>60</td>
<td>26088</td>
<td>26255</td>
<td>15743</td>
<td>0,617</td>
</tr>
<tr>
<td>75</td>
<td>42795</td>
<td>43583</td>
<td>8082</td>
<td>0,368</td>
</tr>
<tr>
<td>90</td>
<td>63000</td>
<td>65000</td>
<td>6500</td>
<td>0,068</td>
</tr>
</tbody>
</table>

Conclusions

The paper presents the method of calculating the mechanical characteristics of multilayer composite panels. The optimal sequence of laying out monolayers was selected. With the help of the developed methodology, the elastic properties of the composite material with different angles of laying layers were determined. Calculations of the mechanical characteristics of carbon fiber panels made of carbon tape and carbon fabric are given, and a comparison of composites made of carbon tape and carbon fabric is made.

References

