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MATHEMATICAL MODEL OF THE AUTOMATIC CONTROL SYSTEM IN THE PROBLEM OF GUARANTEED ACCURACY

Ua Розглядається постановка та розв'язання математичної задачі синтезу системи автоматичного керування (САК) гарантованої точності при невизначених збуреннях застосуванням підходу оберненої динамічної системи. На невизначені збурення не накладаються ніякі обмеження, а самі збурення можуть бути довільними. Показана процедура формування структури САК гарантованої точності, яка забезпечує збереження якості (стійкості та якості перехідного процесу) СК при компенсації збурень, та особливості цієї структури. Запропонований спосіб формування коригуючого впливу для компенсації дії збурень як функцію наближення змінної стану САК до границі допустимого значення. Вперше показано, що такій структурі САК гарантованої точності відповідає нова математична структура – алгебро-диференціальне рівняння (для системи з однією степеню свободи), або система таких рівнянь (для системи з декількома степенями свободи).

En The formulation and solution of the mathematical problem of the synthesis of an automatic control system (ACS) of guaranteed accuracy under uncertain disturbances using the inverse dynamic system approach is considered. No restrictions are imposed on uncertain perturbations, and the perturbations themselves can be arbitrary. The procedure for forming the structure of the ACS with guaranteed accuracy, which ensures the preservation of the quality (stability and quality of the transient process) of the ACS during compensation of disturbances, and the features of this structure are shown. The proposed method of forming a corrective influence to compensate for the effects of disturbances as a function of the approximation of the ACS state variable to the limit of the permissible value. For the first time, it is shown that such a structure of ACS with guaranteed accuracy corresponds to a new mathematical structure - an algebraic differential equation (for a system with one degree of freedom) or a system of such equations (for a system with several degrees of freedom).

Introduction

Creation of stabilization and control systems remains an important task of precision instrument engineering. This is due to the wide scope of application of such systems. At the same time, the issue of development of stabilization systems was reduced to the synthesis of systems for specific input influences

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and in given operating conditions [1 – 5]. Providing the necessary output signal under various influences makes it possible to design universal systems [6].

The problem formulation

Let us consider the possibility of generalization and mathematical formalization of the problem of synthesis of an automatic control system of guaranteed accuracy under uncertain disturbances. Let us determine the mathematical structure of the problem of synthesis of the automatic control system of guaranteed accuracy and the mathematical structure of the problem of forming a corrective influence to compensate for the action of disturbances to fulfill the condition

$$|y| \le y_{ad}\,, \tag{1}$$

where y – the state variable of the control object (output signal),

y_{ad} – admissible value of the object's state variable under the action of a disturbance.

We consider that current disturbances are limited only by their magnitude, which is determined by the possibility of their compensation by executive bodies of the ACS.

Let's consider the formation of a corrective action on the control object as a function of the approximation of the state variable to the limit of the permissible value.

Let's build a control system that would correspond to the equation

$$\Phi(y) = \frac{W(y)}{1 + K(y)} g\,, \tag{2}$$

where $\Phi(y$ - control system transfer function according the disturbance,

$W(y)$ - control object transfer function, $K(y)$ - the coefficient of the corrective effect, depending on the initial value, g - is the disturbance.

Algorithm of formation of the control system structure

Ensuring the quality of the control or stabilization system in many cases requires achieving a certain speed and required accuracy.

Consider a closed control system with a control object W_0 and feedback loop W_c (Fig. 1), where y(t) – the state variable of the object (adjustable value), g(t) – disturbance, y_c – control.

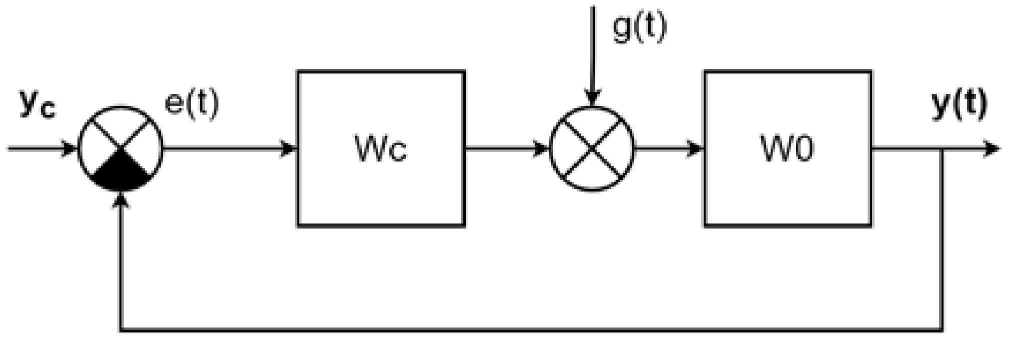


Fig. 1. Control system

No requirements are imposed on the control object. It can be an object of any structure, including a pre-synthesized optimal system.

We will take the transfer function of the control object as

$$W_0 = \frac{K_0}{\sum_{i=0}^{2} a_i s^i}, \quad (3)$$

where K_0 – transfer coefficient of the control object,

$\Sigma a_i s^i$ - its characteristic polynomial.

The optimal transfer function of the regulator is found in the form (4) [7]:

$$W_c(s) = s^{-1}\sum_{i=0}^{3} b_i s^i = s^{-1}H_c(s) = s^{-1}H_{opt}(s), \quad (4)$$

where $b_i s^i = H_c(s) = H_{opt}(s)$ – the characteristic polynomial of the regulator, optimal in terms of speed and accuracy.

The control transfer function accepts the value

$$W_y^{y_c}(s) = \frac{W_0 W_c}{1 + W_0 W_c} = \frac{K_0 s^{-1}\sum_{i=0}^{3} b_i s^i}{\sum_{i=0}^{2} a_i s^i + K_0 s^{-1}\sum_{i=0}^{3} b_i s^i}. \quad (5)$$

Provided that

$$K_0 b_i >> a_{i-1}$$

for i = 1…3, the control transfer function accepts the value $W_y^{y_c}(s) \approx 1$.

The transfer function according the disturbance will have the form

$$W_y^g(s) = \frac{W_0}{1 + W_c W_0}. \quad (6)$$

In accordance with (4) the closed system with transfer functions (5) and (6) will be optimized for speed and accuracy [7].

In optimal systems of guaranteed accuracy under conditions of arbitrary disturbances, the required predetermined accuracy can be provided by introducing an additional link with a transfer function W_a, which is the inverse of W_y^g. It is easy to see that in order to achieve structure (2) by perturbation, it is necessary to provide

$$W_c = KW_0^{-1}. \quad (7)$$

Thus, the feedback controller (control law) must be a K-fold inverse dynamic model of the control object W_0 [8 … 10]. Expressions (6) and (7) are valid for objects that can have an inverse dynamic model. So,

$$W_a(s) = \left[W_y^g(s)\right]^{-1}.$$

In accordance with (7) and fig. 1, we get a structural diagram of the disturbance control system to ensure the specified guaranteed accuracy (fig. 2), where $y(t)$ – controlled state variable, $g(t)$ – disturbance, W_y^g – optimized transfer function of the control object (fig. 1) according to the disturbance, K – is the adjustment coefficient.

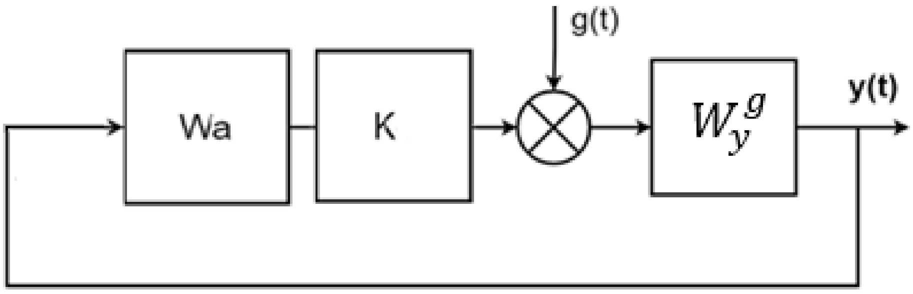


Fig. 2. Control system to ensure guaranteed accuracy

For this system, we will obtain the transfer function of the stabilization system according the disturbance

$$W_1^g(s) = \frac{W_y^g(s)}{1+K}, \quad (8)$$

and

$$y(t) = \frac{W_y^g}{1+K} g(t). \quad (9)$$

It follows from expressions (8) and (9) that it is possible to reduce the output signal of the system from the action of the disturbance by 1+K times, without changing the properties of the pre-synthesized control system. Thus, if the coefficient K is formed as a certain function of the state variable of the system during its observability (output signal), it is possible to adjust the value of the state variable under the action of a disturbance and ensure that it does not exceed the predetermined value (1).

Mathematical formalization of the synthesis problem

Based on (9), the mathematical model of the control system for the synthesis of the system with guaranteed accuracy can be represented in the form

$$y(t)[1+K]W_a(s=\frac{d}{dt})=g(t). \qquad (10)$$

It is not difficult to see that the structure of equation (10) is a new class of equations that can be called algebraic differential nonlinear equations. It can be argued that a control system corresponding to structure (10) will be stable and feasible if:

– a control object is stable W_a .
– the algebraic polynomial $K(y)$ is positive for all $y(t)$.

The specific form of the polynomial K(y) can take into account the possibility of its absence ($K(y)$=0) at time intervals where there is no external influence, or such that the condition is not violated (1) $|y|\leq y_{ad}$.

The obtained structure (10) of the control system allows to study the accuracy of control under the influence of arbitrary disturbances and the selected structure of the control coefficient.

Mathematical simulation

To investigate the effectiveness of the introduction of the adjustment coefficient in the inverse dynamic model of the control system to ensure the specified (guaranteed) accuracy, we will conduct mathematical modeling of the dynamic system (10) for different structures. We will take the control object for modeling in the form (3).

In Fig. 3 shows the response of the system to constant disturbances of various magnitudes in the absence of an algorithm for ensuring control accuracy. The magnitude of the error is proportional to the magnitude of the disturbance.

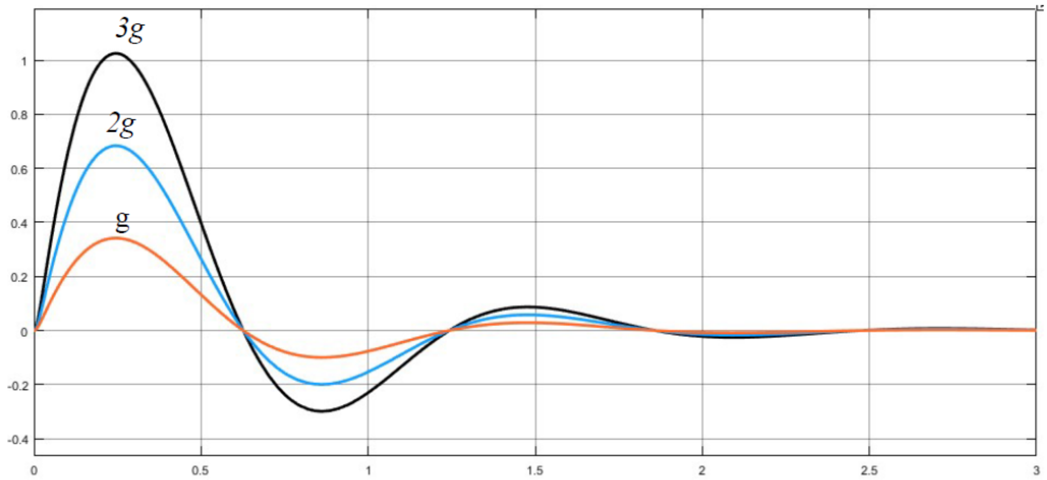


Fig. 3. Response of the system to various disturbances (g, $2g$, $3g$)

Having accepted the permissible value of the output signal y_{ad} = 0,4, we see that the system exceeds it for perturbations of 2g and 3g.

Let the adjustment coefficient be chosen in the form

$$K = \frac{1}{y_{ad} - |y|}, \quad \text{and} \quad K = \frac{1}{e^{y_{ad} - |y|}}.$$

Reaction of the system to a constant disturbance g at y_{ad} = 0,4 shown in Fig. 4.

Fig. 4 shows the efficiency of applying the guaranteed accuracy algorithm. The efficiency of the algorithm can be increased by forming the adjustment coefficient in the form $K = \frac{n}{y_{ad} - |y|}$ and $K = \frac{n}{e^{y_{ad} - |y|}}$ (Fig. 5).

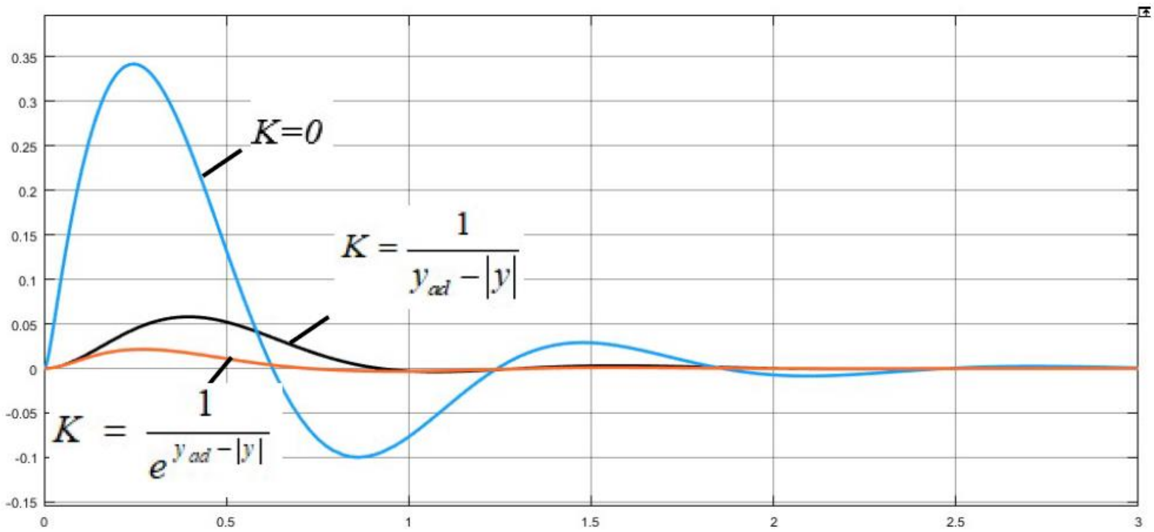
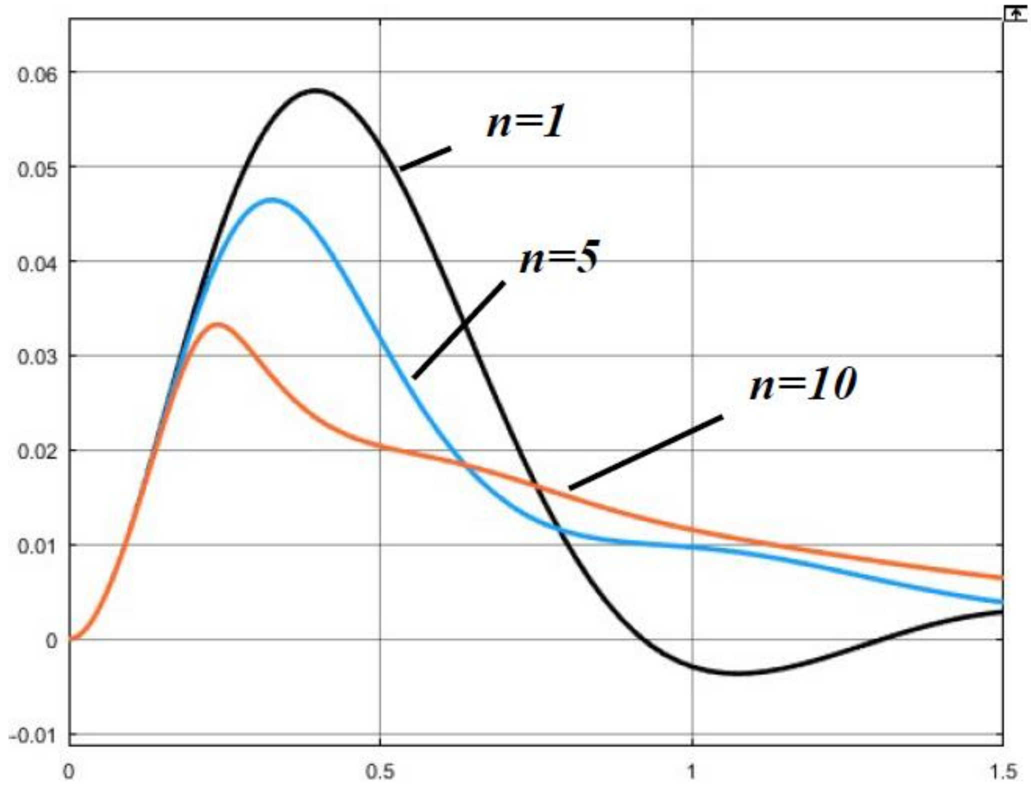
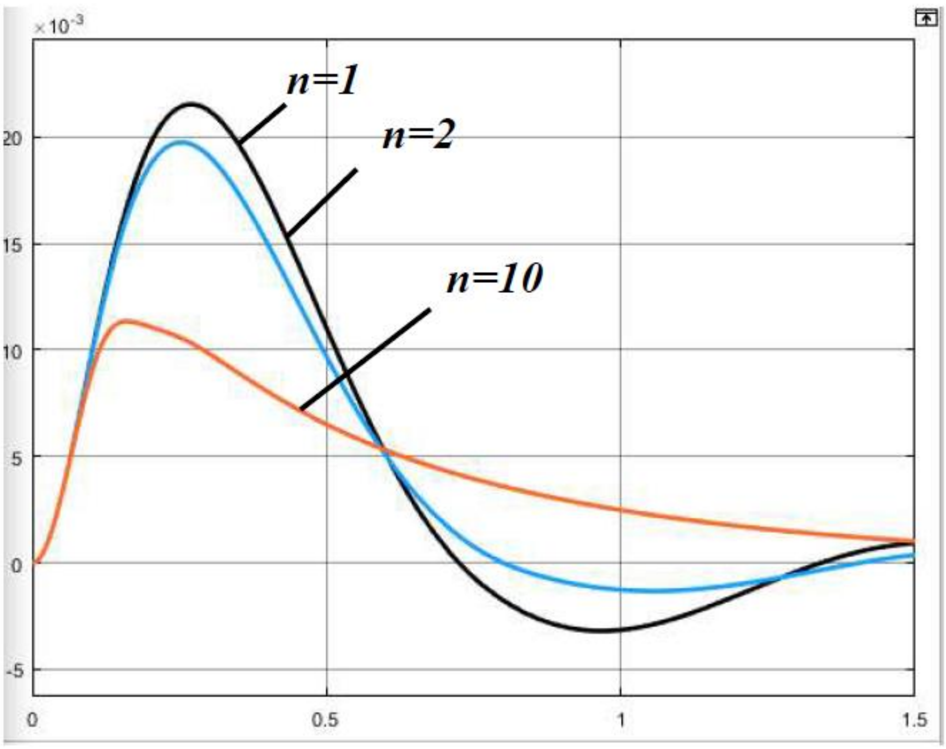


Fig. 4. Output signal of the system without (K=0) and with adjustment coefficients

We can see from Fig. 5, that the use of the function $\frac{1}{e^{y_{ad} - |y|}}$ for the formation of the adjustment coefficient is more effective. The response of the system to disturbances of various magnitudes is shown in Fig. 6, with random disturbances of the ‘white noise’ type – in Fig. 7. with harmonic disturbances in a wide spectrum of frequencies - in Fig. 8.



a)



b)

Fig. 5. Dependence of the output signal on the numerical value of the coefficient n: $K = \dfrac{n}{y_{ad} - |y|}$ (*a*); $K = \dfrac{n}{e^{y_{ad} - |y|}}$ (*b*)

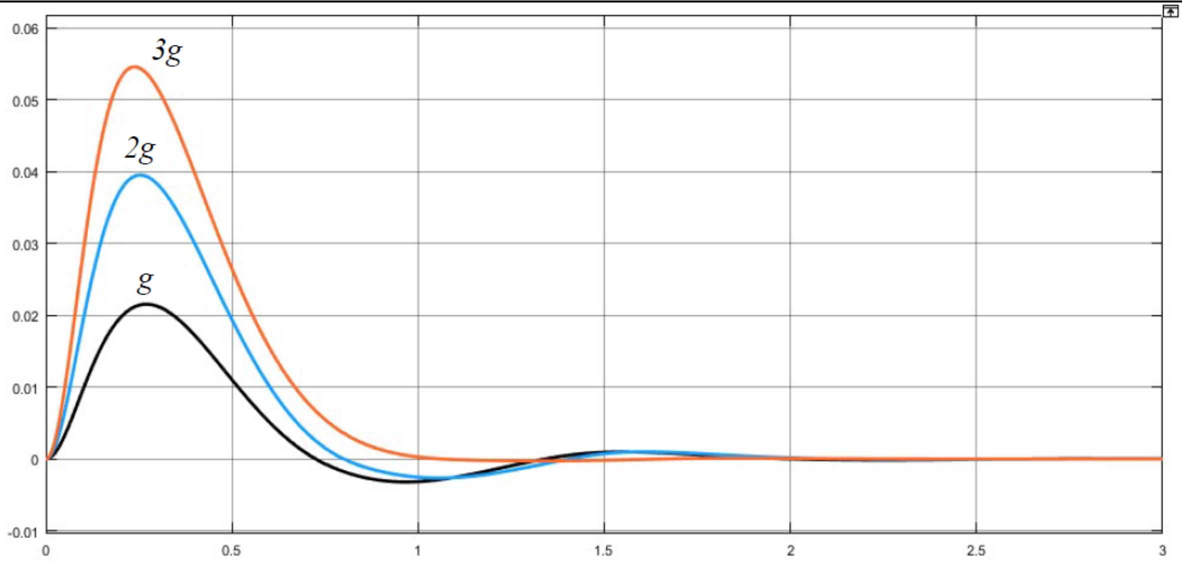


Fig. 6. Output signal of the system at different values of the disturbing signal g and $y_{ad} = 0{,}4$

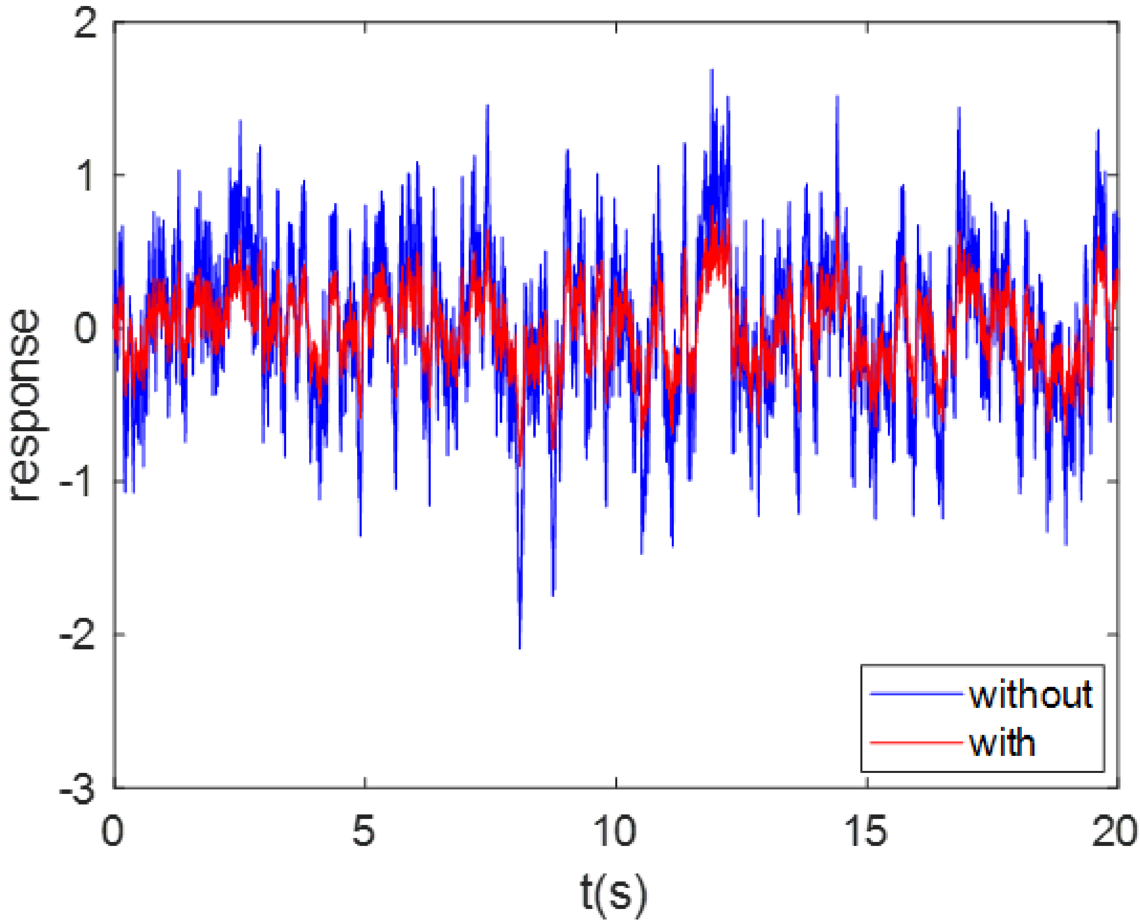


Fig. 7. Output signal of the system with "white" noise and $y_{ad}=1$

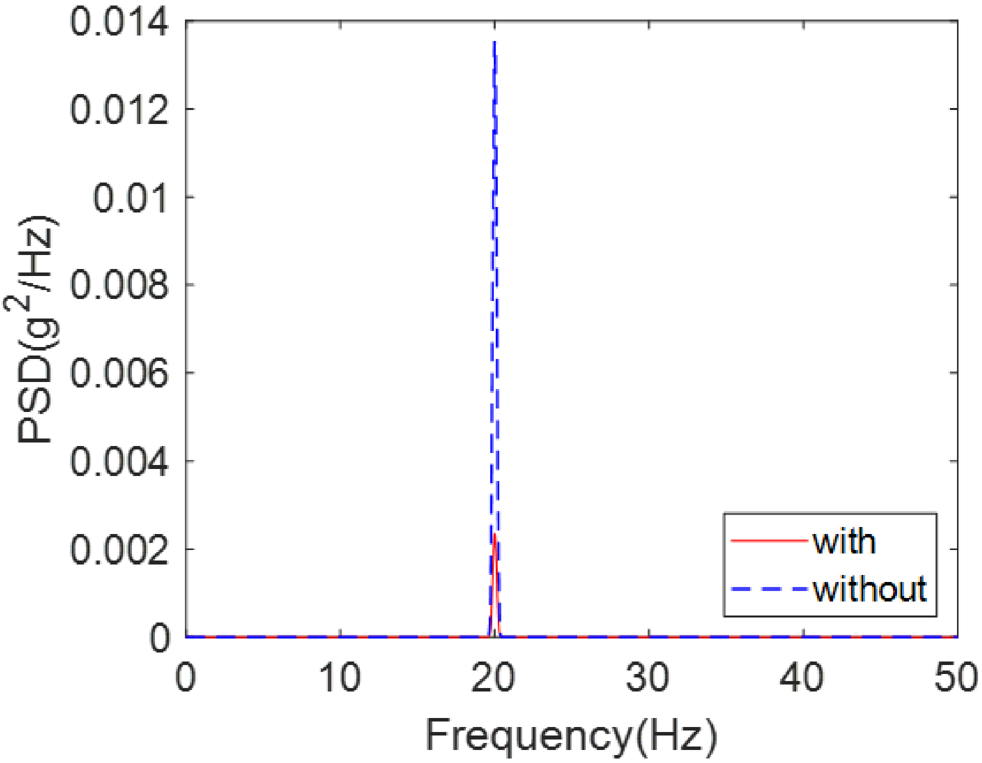


Fig. 8. Amplitude-frequency characteristics of the system with and without the accuracy guarantee algorithm

The simulation results (Fig. 4 ... Fig. 8) show the effectiveness of the proposed method for the synthesis of ACS with guaranteed accuracy. The values of the system state variable do not exceed the permissible value. The variability of the selection of the functions of the adjustment coefficient and the values of its numerical coefficients makes it possible to create universal ACS's with a given guaranteed accuracy.

Conclusions

The problem of mathematical formalization of the synthesis of a guaranteed-accuracy ACS under uncertain disturbances using an inverse dynamic model, which allows investigating the properties of an arbitrary structure SAC under various control algorithms, has been solved.

The proposed method of forming a corrective influence to compensate for the effects of disturbances as a function of the approximation of the ACS state variable to the limit of the permissible value.

The procedure for forming the SAC structure with guaranteed accuracy is shown. It is demonstrated that the structure of the algorithm of guaranteed accuracy ensures the preservation of the quality (stability and quality of the transient process) of the ACS during the compensation of disturbances.

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